Tensor trains and moment conservation for multivariate aggregation in population balance modeling

Robin Ahrens[1] Sabine Le Borne[2]

We consider the numerical solution of the multivariate aggregation population balance equation on a uniform tensor grid. This class of equations is numerically challenging to solve - the computational complexity of "straightforward" algorithms grows exponentially with respect to the number of internal coordinates describing particle properties.
Here, we develop algorithms which reduce the storage and computational complexity to almost linear order, $\mathcal{O}(dn)$ and $\mathcal{O}(dn\log(n))$, respectively, where d denotes the number of internal coordinates and n the number of pivots per internal coordinate. In particular, we develop fast algorithms in tensor train format to evaluate the multidimensional aggregation integral exploiting fast Fourier transformation for the underlying convolution. A further significant result lies in the conservation of the first 2^d moments for our proposed method. Numerical tests confirm the favorable theoretical results concerning computational complexity and conservation of moments.

[1] https://epubs.siam.org/doi/abs/10.1137/090752286
[2] https://www.sciencedirect.com/science/article/pii/S0377042718300840
[3] R Ahrens, S. Le Borne "Tensor trains and moment conservation for multivariate aggregation in population balance modelingSSubmitted 2019

[1]Hamburg University of Technology, Institute of Mathematics
[2]Hamburg University of Technology, Institute of Mathematics