Low-rank tensor methods for PDE-constrained optimization with isogeometric analysis

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Isogeometric analysis (IgA) is a popular method for the discretization of partial differential equations motivated by the use of NURBS (Non-uniform rational B-splines) for geometric representations in industry and science. In IgA the domain representation as well as the discrete solution of a PDE are described by the same global spline functions. However, the use of an exact geometric representation comes at a cost. Due to the global nature and large overlapping support of the basis functions, system matrix assembly becomes especially costly in IgA. To reduce the computing time and storage requirements low-rank tensor methods have become a promising tool. We successfully constructed a framework applying low rank tensor train calculations to IgA to efficiently solve PDE-constrained optimization problems on complex three dimensional domains without assembly of the actual system matrices. The method exploits the Kronecker product structure of the underlying spline space, reducing the three dimensional system matrices to a low-rank format as the sum of a small number of Kronecker products $M = \sum_{i=1}^{n} M_i^{(1)} \otimes M_i^{(2)} \otimes M_i^{(3)}$, where n is determined by the chosen size of the low rank approximation. For assembly of the smaller matrices $M_i^{(d)}$ only univariate integration in the corresponding geometric direction d is performed, thus significantly reducing computation time and storage requirements. The developed method automatically detects the ranks for a given domain and conducts all necessary calculations in a memory efficient low-rank tensor train format. We present the applicability of this framework to efficiently solve large scale PDE-constrained optimization problems as well as an extension to statistical inverse problems using the iterative AMEn block solve algorithm which preserves and exploits the low rank format of the system matrices.

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