Leja and $\Re$-Leja points for computing and preconditioning linear systems

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We introduce Leja sequences on the unit disk $\mathcal{U}$ and $\Re$-Leja sequences on the interval $[-1,1]$ and discuss the growth of associated Newton polynomials and their derivatives and the Lebesgue constant for polynomial interpolation. These growths show that such sequences provide nested alternatives to roots of unity on $\mathcal{U}$ and Tchebychev abscissas on $[-1,1]$ for polynomial interpolation and polynomial preconditioning. In light of these results, given a matrix A with spectrum in a general compact connected domain K of $\mathbb{C}$ or $\mathbb{R}$, we discuss the convergence, stability and accuracy properties of operating with the matrix $f(A)$ on a vector v, i.e. computing $w = f(A)v$, and of Tchebychev-type methods for acceleration and preconditioning Richardson's iteration method. Such problem are discussed e.g. in [TalEzer89,FischerReichel88].
We have studied Leja and $\Re$-Leja sequences in the framework of uni-variate and multi-variate hierarchical polynomial interpolation, see [Chkifa13,Chkifa19]. The particular ordering of such sequences conveys them remarkable structural properties which imply minimal cost, high stability and controlled accuracy in the aforementioned framework. We present here the ramifications of such results in the separate framework of acceleration techniques of linear systems.

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