Prescribing convergence behavior of block Arnoldi and GMRES

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Block Krylov subspace methods are iterative methods for solving systems of linear equations with multiple right-hand sides. At each step, all solutions to the system are sought in the space containing the contribution of each individual right-hand side, which significantly reduces the iteration count compared to solving the systems sequentially (one by one). We focus on block methods for non-Hermitian systems, in particular on block GMRES. While it is known that any non-increasing convergence curve is possible for standard GMRES with one right-hand side and a matrix with a given spectrum, no analog of this result is currently available for block methods, when multiple systems are solved at once.

Using a recent framework for studying these methods as being a single linear system over a *-ring of complex matrices developed by Frommer et al., we show what convergence behavior is admissible for block GMRES and how the matrices and right-hand sides producing such behavior can be constructed. Exploiting the theory of block companion matrices and polynomials with matrix coefficients, we show that the Ritz values obtained by the block Arnoldi method can be fully independent of the eigenvalues of the system matrix and their convergence almost fully independent of the convergence of block GMRES. We demonstrate that in certain sense, the eigenvalues of the coefficient matrix are even less indicative regarding the residual convergence behavior of block GMRES than they are in the standard case.

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