Saddle-point Problems in Liquid Crystal Modelling

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Although the mathematical theory of liquid crystals has been extensively studied for over 75 years, to date there has been much less work done on the numerous interesting and important numerical analysis issues which the study of such materials raises. The focus of this work is on the iterative solution of saddle-point problems which occur frequently, and in multiple ways, in liquid crystal numerical modelling. For example, saddle-point problems arise whenever director models are implemented, through the use of Lagrange multipliers for the pointwise unit vector constraints (as opposed to using angle representations). In addition, saddle-point systems arise when an electric field is present that stems from a constant voltage, irrespective of whether a director model or a tensor model is used, or whether angle representations for directors or componentwise representation with pointwise unit vector constraints is employed. Furthermore, the combination of these two situations (that is, a director model using components, associated constraints and Lagrange multipliers, together with a coupled electric field interaction) results in a double saddle-point structure which presents a particular challenge in terms of numerical linear algebra. In this talk we will present some examples of the saddle-point systems which arise in liquid crystal modelling and discuss their efficient solution using appropriate preconditioned iterative methods with the aim of giving an insight into this new and exciting research area at the interface between liquid crystal theory and numerical analysis.

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