The total least squares and the core problem in tensor settings

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The core problem theory was developed to understand the possible nonexistence of a solution of the linear approximation problem $Ax \approx b$, $b \notin \mathrm{range}(A)$ in the total least squares (TLS) sense. When there is a need to solve, e.g., several linear approximation problems with the same system matrix and parameter(s) dependent right-hand side vectors, it might be advantageous to reformulate it with a right-hand side matrix consisting of individual vectors, $AX \approx B$, where $B \in \mathbb{R}^{m\times d}$. The matrix variant of the TLS minimization motivated a generalization of the core problem. The matrix variant of core problem, contrary to the vector case, still may not have a TLS solution, which raises a natural question why.
This question lead to an effort to study further cases using more general objects, i.e., tensors. In order to better understand the matrix case and to see it in a wider context, we show how to extend the concepts into:

- Tensor right-hand side problems $A \times_1 \mathcal{X} \approx \mathcal{B}$.
- Problems with generalized models, in particular $A_{\mathfrak{L}} X A_{\mathfrak{R}} \approx B$.

With the use of tools of multilinear algebra, we explain the relations of problems formulated with tensors and their matricized variations and mention some computation techniques. We point out similarities and differences concerning the existence and uniqueness of the solution.

[1] https://www.sciencedirect.com/science/article/pii/S0024379518303008
[2] https://www.sciencedirect.com/science/article/pii/S0024379519301703

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