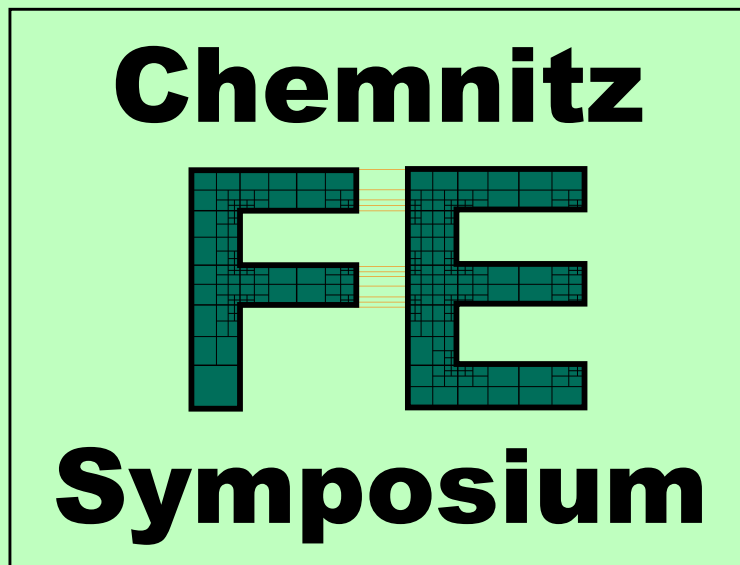


TECHNISCHE UNIVERSITÄT  
CHEMNITZ

Fakultät für Mathematik

# Chemnitz FE-Symposium 2018



Programme

Collection of abstracts

List of participants

Chemnitz, September 24 - 26, 2018

## Scientific Topics:

The symposium is devoted to all aspects of finite elements and related methods for solving partial differential equations.

The topics include (but are not limited to):

- Scientific Computing,
- Mechanics/Applications,
- Inverse Problems,
- Optimization with PDEs,
- Uncertainty Quantification.

This year we particularly encourage talks on:

- Uncertainty Quantification,
- Optimal Control,
- Preconditioning.

## Invited Speakers:

**Howard Elman (University of Maryland, College Park, USA)**

**Lars Ruthotto (Emory University, Atlanta, USA)**

**David Silvester (University of Manchester, UK)**

## Scientific Committee:

Th. Apel (München), S. Beuchler (Hannover), O. Ernst (Chemnitz), G. Haase (Graz),  
H. Harbrecht (Basel), R. Herzog (Chemnitz), M. Jung (Dresden), U. Langer (Linz),  
A. Meyer (Chemnitz), O. Rheinbach (Freiberg), A. Rösch (Duisburg-Essen),  
O. Steinbach (Graz), M. Stoll (Chemnitz)

## Organising Committee:

O. Ernst, R. Herzog, M. Stoll, R. Springer,  
R. Unger, M. Pester, A.-K. Glanzberg, K. Seidel

[www.chemnitz-am.de/cfem2018/](http://www.chemnitz-am.de/cfem2018/)



### Internet Access

The hotel as well as the conference venue offers free internet access. Access details can be obtained from the hotel reception and the registration desk, respectively.

### Food

The conference fee includes:

- Lunch on all three days of the symposium

***Drinks are at your own expense***

- Tea, coffee, soft drinks and snacks during breaks
- Conference dinner on Monday.

For participants staying at the “Biendo Hotel” there is a breakfast buffet from 6 am up to 10 am.

### Conference Dinner

The conference dinner will start on Monday at 7 pm in the restaurant “Malula”.

***One*** soft drink/beer/glass of wine included.

We will meet at 6:40 pm in front of the “Biendo Hotel” and walk to the restaurant together. If you would like to go on your own, the address of the restaurant is:

**Georgstraße 21, 09111 Chemnitz**

### Excursion

The excursion will take place on Tuesday. We will meet at 2 pm in front of the “Biendo Hotel”. After taking the conference photo in the park opposite to the hotel, the excursion will go to the Zeisigwald, a local recreational area in Chemnitz. The walking distance there is approximately 10 km. For the way to the Zeisigwald and back we will use public transport. Back in Chemnitz city center, you can explore the various culinary offers on your own.



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Programme

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## Programme for Monday, September 24, 2018

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	Goal-oriented Adaptivity for Elliptic PDEs with Parametric or Uncertain Inputs		
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	<b>Uncertainty I</b>	<b>CFD I</b>	
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	Multilevel Quadrature for Elliptic Problems on Random Domains by the Coupling of FEM and BEM	High-Order Finite Elements, Pressure-Robustness and Incompressible Generalised Beltrami Flows	
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	Linear Thermoelasticity of Short Fibre Reinforced Composites	The Analogue of grad-div Stabilization in DG Methods for Incompressible Flows: Limiting Behavior and Extension to Tensor-Product Meshes	
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**Programme for Monday, September 24, 2018** (continued)

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<b>Advanced Discretization Techniques</b>		<b>Space-Time FEM</b>	
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17:30	<b>Boniface Nkemzi</b> ..... 34 P- and hp-Versions of the Finite Element Method for Poisson's Equation on Polygonal Domains	34	<b>Hauke Sass</b> ..... 38 Space-Time Trace-FEM for Solving PDEs on Evolving Surfaces
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11:35	<b>Ming Zhou</b> ..... 60 On Block-Krylov Subspace Iterations and the AMP Eigensolver Software	<b>Arnd Rösch</b> ..... 65 FE Error Estimates for Semilinear Parabolic Control Problems in the Absence of the Tikhonov Term
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13:00	<i>Lunch</i>	



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## Collection of Abstracts

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## Multigrid Methods for Stochastic Partial Differential Equations

Howard Elman<sup>1</sup> Tengfei Su<sup>2</sup>

We study a multigrid method for solving large linear systems of equations with tensor product structure. Such systems are obtained from stochastic finite element discretization of stochastic partial differential equations such as the steady-state diffusion problem with random coefficients. When the variance in the problem is not too large, the solution can be well approximated by a low-rank object. In the proposed multigrid algorithm, the matrix iterates are truncated to low rank to reduce memory requirements and computational effort. The method is proved convergent with an analytic error bound. Numerical experiments show its effectiveness in solving the Galerkin systems compared to the original multigrid solver, especially when the number of degrees of freedom associated with the spatial discretization is large.

References:

[1] <https://www.cs.umd.edu/users/elman/papers/Elman-Su.pdf>

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# Adaptive Stochastic Galerkin FEM driven by Two-Level/Hierarchical Error Estimators for Elliptic Parametric PDEs

Michele Ruggeri<sup>1</sup> Alex Bespalov<sup>2</sup> Dirk Praetorius<sup>3</sup> Leonardo Rocchi<sup>4</sup>

We present an adaptive stochastic Galerkin finite element method for a class of parametric elliptic boundary value problems. The adaptive algorithm is steered by a reliable and efficient *a posteriori* error estimator, which can be decomposed into a two-level spatial estimator and a hierarchical parametric estimator [1]. Following [2, 3], the structure of the estimated error is exploited by the algorithm to perform a balanced adaptive refinement of the spatial and parametric discretizations. The adaptive algorithm is proved to be convergent in the sense that the estimated error converges to zero. Numerical experiments underpin the theoretical findings and show that the proposed adaptive strategy helps to mitigate the ‘curse of dimensionality’ which usually afflicts the numerical approximation of parametric PDEs.

## References:

- [1] A. Bespalov, D. Praetorius, L. Rocchi, and M. Ruggeri, *Goal-oriented error estimation and adaptivity for elliptic PDEs with parametric or uncertain inputs*, arXiv:1806.03928 (2018).
- [2] A. Bespalov and L. Rocchi, *Efficient adaptive algorithms for elliptic PDEs with random data*, SIAM/ASA J. Uncertain. Quantif., 6 (2018), 243–272.
- [3] A. Bespalov and D. Silvester, *Efficient adaptive stochastic Galerkin methods for parametric operator equations*, SIAM J. Sci. Comput., 38 (2016), A2118–A2140.

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## Goal-oriented Adaptivity for Elliptic PDEs with Parametric or Uncertain Inputs

Alex Bespalov<sup>1</sup> Dirk Praetorius<sup>2</sup> Leonardo Rocchi<sup>3</sup> Michele Ruggeri<sup>4</sup>

In this talk, we present a goal-oriented adaptive algorithm for approximating linear quantities of interest derived from solutions to elliptic partial differential equations (PDEs) with parametric or uncertain inputs. Specifically, we consider a class of elliptic PDEs where the underlying differential operator has affine dependence on a countably infinite number of uncertain parameters and employ the stochastic Galerkin finite element method to approximate the solutions to the corresponding primal and dual problems. Our algorithm follows the standard adaptive loop:

SOLVE  $\implies$  ESTIMATE  $\implies$  MARK  $\implies$  REFINE.

Here, the error in the goal functional (e.g., the expectation of the quantity of interest) is *estimated* by the product of computable estimates of the energy errors in Galerkin approximations of the primal and dual solutions. Drawing information from the spatial and parametric contributions to these error estimates, the *marking* is performed by extending the strategy proposed in [3]. Finally, following the methodology developed in [2, 1], a balanced adaptive *refinement* of spatial and parametric components of the approximation space is performed by combining the associated error reduction indicators computed for the primal and dual solutions.

We will discuss the results of numerical experiments that demonstrate the effectiveness of our goal-oriented adaptive strategy for a representative model problem with parametric coefficients and for various quantities of interest (including the approximation of pointwise values).

References:

- [1] A. Bespalov and L. Rocchi, *Efficient adaptive algorithms for elliptic PDEs with random data*, SIAM/ASA J. Uncertain. Quantif., 6 (2018), pp. 243–272.
- [2] A. Bespalov and D. Silvester, *Efficient adaptive stochastic Galerkin methods for parametric operator equations*, SIAM J. Sci. Comput., 38 (2016), pp. A2118–A2140.
- [3] M. Feischl, D. Praetorius, and K. G. van der Zee, *An abstract analysis of optimal goal-oriented adaptivity*, SIAM J. Numer. Anal., 54 (2016), pp. 1423–1448.

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## Bayesian Inversion: Posterior Sampling using Surrogate Models

Simona Domesová<sup>1</sup> Michal Béréš<sup>2</sup> Radim Blaheta<sup>3</sup>

We aim at the use of the Bayesian inverse approach for the identification of parameters of differential equations. As unknown parameters, we can consider boundary conditions, source or material parameters in a domain of interest. The solution of such identification problems typically involves repetitive evaluations of a corresponding discretized boundary value problem using numerical methods, e.g. the finite element method (FEM) or its variants. These computational demands can be reduced using surrogate models. The result of the Bayesian inversion is the estimation of the posterior distribution, i.e. the joint probability distribution of the vector of unknown parameters. Since the posterior probability density function depends on the solution of some parametric boundary value problem, it cannot be expressed analytically or sampled directly. The basic Metropolis- Hastings (MH) algorithm can be used to provide samples from the posterior distribution; however, the sampling process is too costly due to high number of evaluations of the forward problem. We shall show that the delayed acceptance MH algorithm combined with a suitable surrogate model works and reduces substantially the number of required forward problem evaluations. We examined several approaches for the surrogate model construction: the stochastic Galerkin method, the stochastic collocation method, and radial basis functions. This approach was tested on model problems, numerical experiments include the tuning of settings of the sampling algorithm parameters.

### References:

- [1] S. Domesová and M. Béréš, *A Bayesian approach to the identification problem with given material interfaces in the Darcy flow*, International Conference on High Performance Computing in Science and Engineering, Springer International Publishing, 2017. (accepted)
- [2] R. Blaheta, M. Béréš, and S. Domesová, *A comparison of deterministic and Bayesian inverse with application in micromechanics*, Applications of Mathematics, 2018. (submitted)
- [3] J. A. Christen and C. Fox, *Markov chain Monte Carlo using an approximation*, Journal of Computational and Graphical statistics, 14(4), 795–810, 2005.

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# Groundwater Flow Equations with Compound Poisson Distributed Input

Toni Kowalewitz<sup>1</sup>   Oliver Ernst<sup>2</sup>   Hanno Gottschalk<sup>3</sup>

We discuss approaches for the computation of quantities of interest for the solution of flow equations in random media. The conductivity is obtained by smoothing or taking level cuts of a Lévy distributed random field, which may result in much rougher random fields than in the Gaussian case. For the special case of a compound Poisson distribution a product quadrature rule will be used and compared to Monte Carlo and Multilevel Monte Carlo simulations. We investigate confidence intervals for both Monte Carlo estimators and the computational effort needed to achieve a certain mean square error.

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## Multilevel Quadrature for Elliptic Problems on Random Domains by the Coupling of FEM and BEM

Marc Schmidlin<sup>1</sup> Helmut Harbrecht<sup>2</sup>

Elliptic boundary value problems which are posed on a random domain can be mapped to a fixed, nominal domain. The randomness is thus transferred to the diffusion matrix and the loading. This domain mapping method is quite efficient for theory and practice, since only a single domain discretisation is needed. Nonetheless, it is not useful for applying multilevel accelerated methods to efficiently deal with the random parameter. This issues from the fact that the domain discretisation needs to be fine enough in order to avoid indefinite diffusion matrices. To overcome this obstruction, we couple the finite element method with the boundary element method. We verify the required regularity of the solution with respect to the random perturbation field for the use of multilevel methods, derive the coupling formulation, and show by numerical results that the approach is feasible.

References:

[1] H. Harbrecht and M. Schmidlin, *Multilevel quadrature for elliptic problems on random domains by the coupling of FEM and BEM*. ArXiv e-prints arXiv:1802.05966, 2018.

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## Linear Thermoelasticity of Short Fibre Reinforced Composites

Rolf Springer<sup>1</sup>

Lightweight structures became more and more important over the last years. One special class of such structures are short fibre reinforced composites, produced by injection moulding. To avoid expensive experiments for testing the mechanical behaviour of these composites proper material models are needed. Thereby, the stochastic nature of the fibre orientation is the main problem.

In this talk we will look onto the simulation of such materials in a linear thermoelastic setting. This means the material is described by the heat conduction tensor  $\kappa(\mathbf{p})$ , the thermal expansion tensor  $\mathbf{T}(\mathbf{p})$ , and the stiffness tensor  $\mathfrak{C}(\mathbf{p})$ . Due to the production process these occurring material quantities has to been understood as random variables.

The classical approach is to average these quantities and solve the linear thermoelastic deformation problem with the averaged expression. We will present a way how this approach can be extended to achieve better approximations of the solutions and will show some numerical results.

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## A Time Dependent Two-Phase Stokes Problem: Well-Posedness and Space-Time FEM Discretization

Igor Voulis<sup>1</sup>   Arnold Reusken<sup>2</sup>

We consider a time dependent Stokes problem with a prescribed, sharp, moving interface. This problem has discontinuous density and viscosity coefficients, a pressure solution that is discontinuous across an evolving interface and an interfacial force (surface tension). This strongly simplified two-phase Stokes equation is a good model problem for the development and analysis of finite element discretization methods for two-phase flow problems. In view of the unfitted finite element methods that are often used for two-phase flow simulations, we are interested in a well-posed variational formulation of this Stokes interface problem in a Euclidean setting.

We discuss the derivation of such a variational formulation in suitable spaces of divergence free functions, as well as the discontinuous-in-time variational formulation involving the pressure variable for the divergence free constraint. This latter formulation is a natural starting point for a space-time finite element discretization with a Discontinuous Galerkin temporal discretization. By combining this DG time-stepping scheme with a spatial XFEM method, we obtain a fully discrete method which is capable of treating discontinuities.

References:

[1] <https://www.springer.com/de/book/9783642196850>

[2] <https://arxiv.org/abs/1803.06339>

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## High-Order Exactly Divergence-free FEM for Transient Incompressible Flows

Philipp W. Schroeder<sup>1</sup> Gert Lube<sup>2</sup>

In [1] we presented a unified approach to exactly divergence-free inf-sup stable FEM for the time-dependent incompressible Navier-Stokes problem, covering both  $H^1$ - and  $H(\text{div})$ -conforming methods. Basic features are pressure-robustness, i.e. additional gradient fields  $\nabla\psi_h$  in the source term lead to a change  $p_h + \psi_h$  of the pressure. This implies that velocity error estimates are not corrupted by large multiples of the best pressure interpolation error. Moreover, the methods are shown to be semi-robust w.r.t. the Reynolds number  $Re$  if  $u \in L^1(0, T; W^{1,\infty}(\Omega))$ , i.e. the error estimates (including the exponential Gronwall factor) do not explicitly on  $Re$ .

In this talk, we report on our numerical experience with benchmark problems in 2D and 3D vortex dynamics using high-order FEM including homogeneous, decaying turbulence, see [2, 3]. Moreover, we present some first results on attached boundary layer flows. In particular, we will discuss the question of required numerical diffusion.

References:

- [1] P.W. Schroeder, C. Lehrenfeld, A. Linke, and G. Lube, *Towards computable flows and robust estimates for in-sup stable FEM applied to the time-dependent incompressible Navier-Stokes equations*, in: Special Issue of SeMA Journal on VMS methods, 2018.
- [2] P.W. Schroeder and G. Lube, *Pressure-robust analysis of divergence-free and conforming FEM for evolutionary Navier-Stokes flows*, J. Numer. Math., 25 (2017) 4, pp. 249–276.
- [3] P.W. Schroeder and G. Lube, *Divergence-free  $H(\text{div})$ -FEM for time-dependent incompressible flows with applications to high Reynolds number vortex dynamics*, J. Sci. Comput. 75 (2018), pp. 830–858.

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## High-Order Finite Elements, Pressure-Robustness and Incompressible Generalised Beltrami Flows

Philipp W. Schroeder<sup>1</sup>   Alexander Linke<sup>2</sup>

We consider FEM for the time-dependent incompressible Navier–Stokes equations:

$$\begin{aligned} \text{Find } (u, p): (0, t_{end}) &\rightarrow \mathcal{V} \times \mathcal{Q} \text{ s.t.} \\ \partial_t u - \nu \Delta u + (u \cdot \nabla)u + \nabla p &= f, \\ \nabla \cdot u &= 0. \end{aligned}$$

In particular, different aspects of high-order pressure-robust and non-pressure-robust methods with respect to their performance for generalised Beltrami flows are compared. This comparison is done both theoretically (error estimates) and practically (numerical examples).

For brevity, error estimates are done in the  $H^1$ -conforming case whereas numerical experiments are performed using  $H(\text{div})$ - and  $L^2$ -conforming Discontinuous Galerkin discretisations.

A generalised Beltrami flow is characterised by the property that the convective term has a scalar potential; that is, there is a sufficiently smooth  $\psi$  such that  $(u \cdot \nabla)u = \nabla\psi$ . However, even for flows where this is not strictly true everywhere in the domain, we demonstrate that practically relevant flow *at least locally* can behave like a generalised Beltrami flow.

Due to the additional irrotational inertia force, pressure-robust methods have certain advantages which can lead to drastically less expensive discretisations compared to using non-pressure-robust methods.

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# The Analogue of grad-div Stabilization in DG Methods for Incompressible Flows: Limiting Behavior and Extension to Tensor-Product Meshes

Mine Akbas<sup>1</sup> Dr. Alexander Linke<sup>2</sup> Prof. Dr. Leo G. Rebholz<sup>3</sup> Philipp W. Schröder<sup>4</sup>

Grad-div stabilization is a classical remedy in conforming mixed finite element methods for incompressible flow problems, for mitigating velocity errors that are sometimes called poor mass conservation. Such errors arise due to the relaxation of the divergence constraint in classical mixed methods, and are excited whenever the spatial discretization has to deal with comparably large and complicated pressures. In this contribution, an analogue of grad-div stabilization for Discontinuous Galerkin methods is studied. Here, the key is the penalization of the jumps of the normal velocities over facets of the triangulation, which controls the measure-valued part of the distributional divergence of the discrete velocity solution. Our contribution is twofold: first, we characterize the limit for arbitrarily large penalization parameters, which shows that the stabilized nonconforming Discontinuous Galerkin methods remain robust and accurate in this limit; second, we extend these ideas to the case of non-simplicial meshes; here, broken grad-div stabilization must be used in addition to the normal velocity jump penalization, in order to get the desired pressure robustness effect. The analysis is performed for the Stokes equations, and more complex flows and Crouzeix-Raviart elements are considered in numerical examples that also show the relevance of the theory in practical settings.

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## Finite Element Approximation of Second Order PDEs in Non-Divergence Form

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Considered are second-order partial differential equations in a non-divergence form, this is,

$$A: \nabla^2 u = f \quad \text{in } \Omega,$$

in some bounded domain  $\Omega \subset \mathbb{R}^2$ . These kind of equations typically arise as subproblems for the solution of Hamilton-Jacobi-Bellman equations in the context of stochastic optimal control or in the linearization of fully non-linear second-order PDEs. Usually, the coefficients of the matrix  $A$  are non-differentiable in these applications and thus, the equations must be discussed in the non-variational form.

We investigate a non-conforming finite element approximation of these problems using higher-order Lagrange-elements for the approximation of  $u$  and continuous or discontinuous elements for some discrete Hessian. Of particular interest are a priori and a posteriori error estimates.

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## Numerical Solution of Isaacs Equations

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Isaacs equations form a family of fully nonlinear PDEs arising naturally from stochastic zero-sum two player games. We consider the problem of the following form:

$$-\delta_t + \inf_{\beta} \sup_{\alpha} (L^{(\alpha, \beta)} v - f^{(\alpha, \beta)}) = 0,$$

where  $L^{(\alpha, \beta)}$  are first- or second-order linear operators. One can think of them as a generalisation of Hamilton–Jacobi–Bellman equations from optimal control theory. The aim of the talk is to present a novel finite element method for these Isaacs equations. The main difficulty arises from the non-convex structure of the underlying inf-sup operator. The presented numerical method permits fully implicit, semi-explicit and explicit time discretizations, with the fully implicit one being unconditionally stable. We discuss monotonicity, L-infinity stability and consistency, even for unstructured meshes. Based on this we obtain uniform convergence of numerical solutions to the viscosity solution of Isaacs problem.

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## Unsteady Convection Diffusion Equation with Random Input Data

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Partial differential equations (PDEs) with random input data is one of the most powerful tools to model oil and gas production as well as groundwater pollution control. However, the information available on the input data is very limited, which cause high level of uncertainty in approximating the solution to these problems. To identify the random coefficients, the well-known technique Karhunen Loève (K–L) expansion has some limitations. K–L expansion approach leads to extremely high dimensional systems with Kronecker product structure and only preserves two-point statistics, i.e., mean and covariance. To address the limitations of the standard K–L expansion, we propose Kernel Principal Component Analysis (PCA).

In this talk, we investigate the numerical solution of unsteady convection diffusion equation with random input data by using stochastic Galerkin method. Since the local mass conservation play a crucial role in reservoir simulation and transport problem, we use discontinuous Galerkin method for the spatial discretization. On the other hand the rational deferred correction method is performed for the temporal discretization. We provide some numerical results to illustrate the efficiency of the proposed approach.

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# Stochastic Mixed FEM for Parameter-Dependent Nearly Incompressible Elasticity Equations

Arbaz Khan<sup>1</sup> Catherine Powell<sup>2</sup> David J. Silvester<sup>3</sup> Alex Bespalov<sup>4</sup>

In this talk, we present some recent developments in the theory and implementation of mixed formulations of parameter dependent nearly incompressible linear elasticity problems. We introduce a novel three-field mixed variational formulation of the PDE model and discuss its approximation by stochastic Galerkin mixed finite element techniques. A priori and a posteriori error analysis will be presented. Numerical results will also be presented to validate the theory.

References:

[1] <https://arxiv.org/abs/1803.01572>

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## Fluid-Structure Interactions with Contact using Nitsche's Method

Stefan Frei<sup>1</sup> Erik Burman<sup>2</sup> Miguel A. Fernández<sup>3</sup>

In this presentation we develop a Nitsche-based contact formulation for fluid-structure interaction (FSI) problems with contact. Our approach is based on the works of Chouly and Hild for contact problems in solid mechanics. Using a suitable extension of the fluid equations below the contact surface, we are able to formulate the FSI interface and the contact conditions simultaneously in equation form on a joint interface-contact surface  $\Gamma(t)$ . Due to the continuous switch between interface and boundary conditions, the so-called “chattering” phenomenon known in the engineering literature, is prevented. We show a stability result and present some numerical examples to investigate the performance of the method.

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## Different Inlet Boundary Conditions for Fluid-Structure Interaction Problem Approximated by FEM

Jan Valášek<sup>1</sup> Petr Sváček<sup>2</sup>

The contribution deals with the numerical simulation of fluid-structure interaction problem, here represented by the human vocal folds vibration excited by the fluid flow. The main attention is paid to the inlet boundary conditions. The classical Dirichlet boundary condition in the form of prescribed velocity has the drawback of unphysical pressure increase during channel closing phase at each vocal folds vibration cycle. The another often used possibility is to prescribe pressure drop between inlet and outlet by the do-nothing type of boundary condition. It usually leads to quite high oscillation of inlet velocity. In order to overcome these disadvantages, the penalization approach is investigated, where beside the given pressure drop the inlet velocity is weakly enforced with the aid of the penalization term. This is quite original approach within scope of finite element method, although usual approach within discontinuous Galerkin methods. The vocal folds are modelled as an elastic isotropic body with assumption of small displacements. Due to the small velocities compared to the speed of sound the fluid flow can be described by the incompressible Navier-Stokes equations. For the purpose of numerical simulation of the time varying computational domain the arbitrary Lagrangian-Euler method is applied. The whole problem is solved by the finite element method based solver.

Numerical results will be presented and analyzed.

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## Hydrostatic Reconstructed Force Scheme and Application to the non unique Riemann Solutions for the Shallow Water Equations

Tim Haubold<sup>1</sup>

The well known shallow water equations are system of hyperbolic equations, for which a Riemann problem can be defined. For a general hyperbolic system a Riemann problem can be defined as an initial value problem with piecewise constant initial data with one discontinuity. In case of the shallow water equations, the incorporation of the source term, e.g. the bathymetry, is an ongoing topic. Since the classical fractional step method fails to preserve steady states in case of such a source term, we will take a look at the so called hydrostatic reconstruction introduced by Audusse in 2004.

It has been shown by Andrianov and was later expanded by Han and Warnecke, that the shallow water equations with a Riemann problem can develop multiple solutions. Furthermore Andrianov has shown that different finite volume schemes approach different solutions. We will compare hydrostatic reconstructed schemes, especially Toro's Force scheme, with other *well-balanced* schemes. Using the exact solutions from Han and Warnecke, we take a look at which solutions will be approached.

This is joint work with Ee Han (formerly University of Bremen)

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## Tensor-valued FE-spaces on exactly represented surfaces

Lars Ludwig<sup>1</sup>

In recent years some growing interest has been seen in finite element spaces on surfaces. In particular, the construction of vector- or tensor-valued finite element spaces on manifolds are still a non-trivial and much-discussed problem. In this talk a new direct approach to discretize tensor-valued FE-spaces on isogeometrically represented manifolds is suggested and supported with various numerical tests and simulations.

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## On a New Mixed Formulation of Kirchhoff-Love and Reissner-Mindlin Plates and Shells

Walter Zulehner<sup>1</sup> Katharina Rafetseder<sup>2</sup>

In 2D linear elasticity it is well-known that a stress tensor field satisfying the homogeneous equilibrium equation can be expressed in terms of the Airy stress function provided the domain is topologically simple. There is a similar result for the bending moment tensor field in plate models, if the mid-surface of the plate is simply connected. While the stress tensor field in 2D linear elasticity can be written as a second-order differential operator applied to the Airy stress function, the bending moment tensor field in plate models is only a first-order differential operator applied to some 2D vector field. We will show how this result can be used to reformulate the Kirchhoff-Love and the Reissner-Mindlin plate and shell models as well-posed second-order systems. The reformulation of the plate and shell models as second-order systems allows for discretization methods in approximation spaces with continuous functions. This includes standard continuous Lagrangian finite element methods and spline spaces from isogeometric analysis on multi-patch domains with continuous patching only.

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## Finite Element Approximation of Prestressed Shells

Ismail Merabet<sup>1</sup> Serge Nicaise<sup>2</sup>

This work deals with the finite element approximation of a prestressed shell model. Contrary to the other known shell models, like Koiter's, Naghdi's, Budiansky–Sanders . . . , the considered model is not necessary positive and the prestressed term is predominate. In addition to existence and uniqueness results of solutions of the continuous and discrete problems we derive some a priori error estimates. Numerical tests are given that validate and illustrate our approach.

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## P- and hp-Versions of the Finite Element Method for Poisson's Equation on Polygonal Domains

Boniface Nkemzi<sup>1</sup>    Stephanie Tanekou<sup>2</sup>

We consider boundary value problems for the Poisson equation on polygonal domains with general nonhomogeneous mixed boundary conditions and derive, on the one hand, explicit extraction formulas for the coefficients of the singularities. On the other hand, the formulas are used to construct efficient adaptations for the  $h$ -,  $p$ - and  $hp$ -versions of the finite element method for the numerical treatment. A priori error estimates show that the  $h$ -version of the finite element algorithm exhibits the same rate of convergence as it is known for problems with smooth solutions. However, the principal results of the present work are the robust exponential convergence results for the  $p$ - and  $hp$ -versions of the finite element method on quasiuniform meshes. In fact, it is shown that if the input data (source term and boundary data) are piecewise analytic, then with appropriate choices of conforming finite element subspaces  $V_N$  of dimension  $N \in \mathbb{N}$ , the  $p$ - and  $hp$ -versions of the finite element algorithms on quasiuniform meshes yield approximate solutions  $u_N$  to the exact solution  $u$  that satisfy the estimates  $\|u - u_N\|_{H^1(\Omega)} \leq C_1 e^{-b_1 N^{\frac{2}{3}}}$  and  $\|u - u_N\|_{H^1(\Omega)} \leq C_2 e^{-b_2 N^{\frac{1}{3}}}$ , respectively. Several numerical experiments are included to illustrate the practical effectiveness of the proposed algorithms. The results show that the theoretical error analyses are attained within the range of engineering computations

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## Error Estimates for a Stabilised Space-Time Finite Element Method for the Wave Equation

Marco Zank<sup>1</sup>   Olaf Steinbach<sup>2</sup>

For the discretisation of time-dependent partial differential equations usually explicit or implicit time stepping schemes are used. An alternative approach is the usage of space-time methods, where the space-time domain is discretised and the resulting global linear system is solved at once. In any case CFL conditions play a decisive role for stability.

In this talk the model problem is the scalar wave equation. First, a space-time variational formulation of the wave equation and its discretisation via a space-time approach including a CFL condition are motivated and discussed. Second, to gain a deeper understanding of the CFL condition an ordinary differential equation corresponding to the wave equation is analysed. For this ordinary differential equation an unconditionally stable numerical scheme is introduced. By transferring this idea to the wave equation a stabilised space-time finite element method for the wave equation is presented. For this method stability and error estimates are discussed.

Finally, numerical examples for a one-dimensional and a two-dimensional spatial domain are shown.

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## On Space-Time Variational Formulations for Maxwell's Equations

Julia Hauser<sup>1</sup>   Olaf Steinbach<sup>2</sup>

Maxwell's equations are the key to electro magnetic problems. There are many approaches to solve these equations and most try to eliminate the time derivative to simplify the equations. In contrast to these methods we consider time as another dimension and look at Maxwell's equations in a corresponding 4D space-time setting. For that purpose we look at the equations on a bounded Lipschitz domain in space and a bounded interval in time. The electric permittivity and magnetic permeability shall be symmetric, positive definite and bounded matrix functions. We will consider different variational formulations and try to determine under what conditions Maxwell's equations are uniquely solvable.

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## Space-Time Trace-FEM for Solving PDEs on Evolving Surfaces

Hauke Sass<sup>1</sup> Arnold Reusken<sup>2</sup>

We consider the parabolic equation

$$\begin{aligned} \dot{u} + (\operatorname{div}_{\Gamma(t)} \mathbf{w})u - \nu_d \Delta_{\Gamma} u &= f \quad \text{on } \Gamma(t), \quad t \in (0, T], \\ u(x, 0) &= 0 \quad \text{on } \Gamma(0), \end{aligned}$$

posed on a smooth, closed and evolving surface  $\Gamma(t)$ , which is advected by a given velocity field  $\mathbf{w}$ . Here,  $\dot{u}$  denotes the material derivative. Based on a space-time weak formulation, we present different fully discrete Eulerian finite element methods using DG in time and continuous in space finite elements. The zero level of the space-time linear approximation of the corresponding level set function defines a Lipschitz space-time manifold  $S_h$ , which approximates the evolving surface  $\Gamma(t)$ . Based on volumetric finite elements we use traces on  $S_h$  as trial and test surface FE-spaces for discretization. In a setting with exact and smooth geometry first order error bounds in an energy norm are known to be valid, cf. [1]. In recent work we analyze the effect of the geometry approximation on the discretization error. Furthermore, a space-time surface normal stabilization is introduced that ensures better conditioning properties of the ensuing discretization matrices. In the presentation we explain this space-time Trace-FEM and present results of numerical experiments. Moreover, a few main results of the error analysis are discussed.

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# Variational Time Discretization of Higher-Regularity for the Wave Equation

Mathias Anselmann<sup>1</sup> Markus Bause<sup>2</sup> Uwe Köcher<sup>3</sup>

We present families of variational space-time finite element discretisations for the hyperbolic wave equation, written as the first-order in time system,

$$\partial_t u - v = 0, \quad \partial_t v - \nabla \cdot (c \nabla u) = f$$

and equipped with appropriate initial and boundary conditions. This system is studied as a prototype model for elastic waves with applications in, for instance, fluid-structure interaction or non-destructive material inspection.

Firstly, we introduce a family of  $C^1$  continuous time discretizations based on a computationally cheap post-processing of continuous in time Petrov–Galerkin approximations; cf. [1, 2]. Optimal order error estimates for the fully discrete scheme are given and illustrated by numerical experiments; cf. [1]. Secondly, a class of schemes that combine collocation and variational equations and ensure higher-order regularity in time is presented; cf. [3]. The convergence properties of its members admitting  $C^1$ - and  $C^2$ -regularity in time are analyzed numerically. For a more sophisticated problem of practical interest a comparative study of all schemes is provided.

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# Stochastic Galerkin Approximation of Nearly Incompressible Elasticity

David Silvester<sup>1</sup>

We discuss the key role that bespoke linear algebra plays in modelling PDEs with random coefficients using stochastic Galerkin approximation methods. As a specific example, we consider nearly incompressible linear elasticity problems with an uncertain spatially varying Young's modulus. The uncertainty is modelled with a finite set of parameters with prescribed probability distribution. We introduce a novel three-field mixed variational formulation of the PDE model and focus on the efficient solution of the associated high-dimensional indefinite linear system of equations. Eigenvalue bounds for the preconditioned system are established and shown to be independent of the discretisation parameters and the Poisson ratio.

This is joint work with Alex Bespalov, Arbaz Khan and Catherine Powell

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## **New Finite Elements by using Potential Maps - With Applications to Linear Elasticity**

Martin Neumüller<sup>1</sup>

In this talk we consider simplex-meshes and we use potential maps to construct the local polynomial finite element spaces for different interesting differential operators. These local finite element spaces obtained by this construction principle automatically fulfill the exact sequence property. Moreover we show a technique to easily obtain the local degrees of freedom. We apply this technique to the elasticity complex resulting, in the lowest order case, in conforming linear elements for the symmetric stress tensor. By using regular decompositions we are also able to construct preconditioners by following the work by Hiptmair and Xu.

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## Efficient Preconditioning of hp-FEM Matrices by Hierarchical Low-Rank Approximations

Paolo Gatto<sup>1</sup> Jan S. Hesthaven<sup>2</sup>

In this talk I will introduce a preconditioner based on low-rank compression of Schur complements. The construction is inspired by the well-known nested dissection strategy, and relies on the assumption that the Schur complements that arise in the elimination process can be approximated, to high precision, by compressible matrices. The preconditioner is built as an approximate  $LDM^t$  factorization of a given matrix  $A$ , and no knowledge of  $A$  in assembled form is required by the construction. The  $LDM^t$  factorization is amenable to fast inversion, and the inverse can be applied fast as well. I will present numerical experiments that investigate the behavior of the preconditioner in the context of Discontinuous Galerkin finite element approximations of positive-definite problems, as well as indefinite wave propagation problems.

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## On a Commutator Preconditioner for the Navier-Stokes Equations

Jan Blechta<sup>1</sup>   Josef Málek<sup>2</sup>   Martin Řehoř<sup>3</sup>

We provide a novel analysis of the PCD (pressure convection-diffusion) preconditioner based on infinite-dimensional interpretation of the underlying operators, which is the basis of operator preconditioning theory. Our results help to answer open questions concerning practical implementation and performance. In particular, we clarify the choice of boundary conditions which is, to our knowledge, a problem missing rigorous analysis, although some heuristic treatment has appeared; see [1, 2]. Moreover, our approach provides a unified theory for both variants (order of the commuted operators) which have thus far only been treated separately in the literature. Last but not least, our infinite-dimensional analysis reveals Fredholm structure in the preconditioner which must be respected in the discretization; failure to do so results in significant deterioration of performance, which has previously been observed but not correctly identified and resolved.

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## Low Rank Tensor Methods for PDE-constrained Optimization with Isogeometric Analysis

Alexandra Bünger<sup>1</sup> Martin Stoll<sup>2</sup>

Isogeometric analysis has become one of the most popular methods for the discretization of partial differential equations motivated by the use of NURBS for geometric representations in industry and science. A crucial challenge lies in the solution of the discretized equations, which we discuss in this talk with a particular focus on PDE-constrained optimization subject to a PDE discretized using IGA.

The discretization results in a system of large mass and stiffness matrices, which are typically very costly to assemble. To reduce the computing time and storage requirements low-rank tensor methods as proposed by Mantzaflaris et al. have become a promising tool. We present a framework for the assembly of these matrices  $M$  in low-rank form as the sum of a small number of Kronecker products  $M = \sum_{i=1}^n \otimes_{d=1}^D M_i^{(d)}$ , where  $D$  is the geometry's dimension (2 or 3) and  $n$  is determined by the chosen size of the low rank approximation. For assembly of the smaller matrices  $M_i^{(d)}$  only univariate integration in the corresponding geometric direction  $d$  is required.

The resulting low rank Kronecker product structure of the mass and stiffness matrices can be used to solve a PDE-constrained optimization problem without assembly of the actual system matrices. We present a framework which preserves and exploits the attained Kronecker product format using the *amen block solve* algorithm from the tensor train toolbox in MATLAB to efficiently solve the corresponding KKT system of the optimization problem. We illustrate the performance of the method on various examples.

References:

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# Efficient Multiphysics Preconditioning for Poroelastic Wave Propagation

Uwe Köcher<sup>1</sup> Markus Bause<sup>2</sup>

The higher-order mixed-multiphysics space-time finite element discretisation of the dynamic Biot-Allard equations including elastic wave propagation in a fluid-saturated porous media

$$\partial_t((1 - \phi)\rho_s \mathbf{v} + \phi\rho_f \mathbf{q}) - \nabla \cdot \boldsymbol{\sigma}(\mathbf{u}, p) = \mathbf{f},$$

$$\boldsymbol{\sigma}(\mathbf{u}, p) = \mathbf{C}\boldsymbol{\epsilon}(\mathbf{u}) - p\mathbf{1}, \boldsymbol{\epsilon}(\mathbf{u}) = (\nabla\mathbf{u} + \nabla\mathbf{u}^T)/2, \text{ coupled fluid flow}$$

$$\partial_t(c_0 p + \alpha\nabla \cdot \mathbf{u}) + \nabla \cdot \mathbf{q} = s$$

and  $\mathbf{q} = -\mathbf{K}(\nabla p - \rho_f \mathbf{g} - \partial_t \mathbf{v})$ , equipped with suitable initial and boundary data, results in huge linear systems with a certain block structure. The unknown fields are the displacement  $\mathbf{u}$ , velocity  $\mathbf{v}$ , fluid-pressure  $p$  and fluid flux  $\mathbf{q}$ . We solve the arising linear systems in a fully-coupled monolithic way by applying the flexible GMRES iterative linear system solver. In this contribution we present details on our efficient preconditioning strategy consisting of the application of multiple-step fixed-stress iterations in a multigrid in time setting. The performance of the automatically tuned preconditioning strategy and implementation in our distributed-parallel solver suite DTM++ for the deal.II library is analysed carefully with several challenging numerical experiments with relevance to physical problems.

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- [5] M. Bause, F. Radu and U. Köcher, *Space-time finite element approximation of the Biot poroelasticity system with iterative coupling*, Comput. Meth. Appl. Mech. Engrg. 320, pp. 745–768, 2017.

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# Analysis and Numerics of a Microscale Model for Lithium Ion Batteries

Willy Dörfler<sup>1</sup>

This talk was added to the programme at short notice (without abstract).

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## A Priori Error Estimates in Non-Energy Norms for the Two-Dimensional Signorini Problem

Constantin Christof<sup>1</sup>   Christof Haubner<sup>2</sup>

This talk is concerned with error estimates for the piecewise linear finite element approximation of the two-dimensional scalar Signorini problem on a convex polygonal domain  $\Omega$ . Using a Céa-type lemma, a supercloseness result and a non-standard duality argument, we prove  $W^{1,p}(\Omega)$ -,  $L^p(\Omega)$ -,  $L^\infty(\Omega)$ -,  $W^{1,\infty}(\Omega)$ - and  $H^{1/2}(\partial\Omega)$ -error estimates under mild assumptions on the contact sets of the continuous and the discrete solution. The obtained orders of convergence turn out to be optimal for problems with essentially bounded right-hand sides.

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## Discretization error estimates for Very Weak solutions of elliptic boundary value problems

Thomas Apel<sup>1</sup>   Johannes Pfefferer<sup>2</sup>

The investigation of Dirichlet control problems with  $L^2$ -regularization leads to the consideration of very weak solutions of boundary value problems. We discretize them here with piecewise linear finite elements on graded meshes and analyze the error in various norms. The critical detail is that we cannot use Céa's lemma since the solution is in general not in  $H^1$ . These results close a gap in the analysis of the discretization of Dirichlet control problems on graded meshes.

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## An Adjoint Based Multi-Goal Oriented Error Estimation for Non-linear Problems

Bernhard Endtmayer<sup>1</sup> Ulrich Langer<sup>2</sup> Thomas Wick<sup>3</sup>

In this talk, we formulate goal-oriented mesh adaptivity for multiple functionals of interest for nonlinear problems in which both the Partial Differential Equation (PDE) and the goal functionals may be nonlinear [1]. The presented method is based on a posteriori error estimates in which the adjoint problem is used [3] and a partition-of-unity is employed for the error localization that allows us to formulate the error estimator in the weak form [2]. Finally our techniques are substantiated with a numerical example, where mesh adaptivity is driven by more than one goal functionals.

This work has been supported by the Austrian Science Fund (FWF) under the grant P 29181 'Goal-Oriented Error Control for Phase-Field Fracture Coupled to Multiphysics Problems'. The first author thanks the Doctoral Program on Computational Mathematics at JKU Linz the Upper Austrian Government for the support when starting the preparation of this work. The third author was supported by the Doctoral Program on Computational Mathematics during his visit at the Johannes Kepler University Linz in March 2018.

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## Mesh Refinement for the Adaptive Isogeometric Method

Philipp Morgenstern<sup>1</sup>

We introduce a mesh refinement algorithm for the Adaptive Isogeometric Method using multivariate T-splines. We investigate linear independence of the T-splines, nestedness of the T-spline spaces, and linear complexity in the sense of a uniform upper bound on the ratio of generated and marked elements, which is crucial for a later proof of rate-optimality of the method. Altogether, this work paves the way for a provably rate-optimal Adaptive Isogeometric Method with T-splines in any space dimension.

As an outlook to future work, we outline an approach for the handling of zero knot intervals and multiple lines in the interior of the domain, which are used in CAD applications for controlling the continuity of the spline functions, and we also sketch basic ideas for the local refinement of two-dimensional meshes that do not have tensor-product structure.

References:

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## Deep Neural Networks motivated by PDEs

Lars Ruthotto<sup>1</sup>

One of the most promising areas in artificial intelligence is deep learning, a form of machine learning that uses neural networks containing many hidden layers. Recent success has led to breakthroughs in applications such as speech and image recognition. However, more theoretical insight is needed to create a rigorous scientific basis for designing and training deep neural networks, increasing their scalability, and providing insight into their reasoning.

This talk bridges the gap between partial differential equations (PDEs) and neural networks and presents a new mathematical paradigm that simplifies designing, training, and analyzing deep neural networks. It shows that training deep neural networks can be cast as a dynamic optimal control problem similar to path-planning and optimal mass transport. The talk outlines how this interpretation can improve the effectiveness of deep neural networks. First, the talk introduces new types of neural networks inspired by parabolic, hyperbolic, and reaction-diffusion PDEs. Second, the talk outlines how to accelerate training by exploiting multi-scale structures or reversibility properties of the underlying PDEs. Finally, recent advances on efficient parametrizations and derivative-free training algorithms will be presented.

The talk is joint work with the groups of Eldad Haber and Eran Treister and based in part on [1, 2, 3, 4].

References:

[1] <https://arxiv.org/abs/1705.03341>

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[3] <https://arxiv.org/abs/1709.03698>

[4] <https://arxiv.org/abs/1804.04272>

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# Semi-Supervised Learning on Hypergraphs using Graph Convolutional Networks

Dominik Alfke<sup>1</sup>   Martin Stoll<sup>2</sup>

Graph Convolutional Networks are a special type of Neural Network designed to combine the flexibility of Machine Learning with standard tools from graph theory. In applications that supply adjacency information in addition to data point features, GCNs may incorporate this information by means of the graph Laplacian operator, which is used to define a generalization of convolution on graph-based data. Spectral convolutional filters are learned by training with partially labelled data in a semi-supervised classification context.

We introduce the extension of GCNs to hypergraphs, a generalization of traditional graphs where edges no longer represent connections between two, but an arbitrary number of nodes. We discuss the method's benefits and challenges in the context of this application. We further introduce special spectral filters that are required to reduce the complexity of the layer operations caused by the non-sparse hypergraph Laplacian matrix. We finally present the method's strong practical performance by application to common hypergraph datasets.

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## Reliable and fast Solving of Small-Strain Plasticity Problems with a Nonsmooth Multigrid Method

Patrick Jaap<sup>1</sup>   Oliver Sander<sup>2</sup>

We present a new approach for solving small-strain plasticity problems called the *Truncated Nonsmooth Newton Multigrid Method* (TNNMG). It is based on the primal form of the plasticity problem, where the unknowns are the *displacement* and the *plastic strain*. Time discretization results in a sequence of convex minimization problems. It works both for smooth and nonsmooth yield laws, as well as for various linear and nonlinear hardening rules.

TNNMG is designed to minimize such convex functionals with block-separable nonlinearities by combining local nonlinear smoothing steps and global linear multi grid corrections. TNNMG has been proven to be *globally convergent*, and is extremely fast in practice: With this method we can compute a complete plasticity time step in less time than one classical predictor–corrector step.

In the presentation the TNNMG method will be explained and applied to rate-independent small-strain plasticity theory. Several numerical simulations will be given for *Tresca* and *Von Mises* yield laws as well as for different hardening rules. The efficiency of the algorithm will be compared to the predictor–corrector approach.

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## Patient-Specific Cardiac Parametrization from Eikonal Simulations

Gundolf Haase<sup>1</sup> Daniel Ganellari<sup>2</sup>

Simulations in cardiac electrophysiology use the bidomain equations describing the intercellular and the extracellular electrical potential. Its difference, the trans-membrane potential, is responsible for the excitation of the heart and its steepest gradients form an excitation wavefront propagating in time. This arrival time  $\varphi(x)$  of the wavefront at some point  $x \in \Omega$  can be approximated by the simpler Eikonal equation. The accuracy of these simulations is limited by unavailable patient specific conductivity data. The human heart consists of various tissues with different conductivity parameters. We group these tissues into  $m$  different classes with its individual scaling parameter  $\gamma_k$  yielding to the modified Eikonal equation

$$\sqrt{(\nabla\varphi(x, \underline{\gamma}))^T \gamma_k \cdot M(x) \nabla\varphi(x, \underline{\gamma})} = 1 \quad x \in \Omega_k$$

where the velocity information  $M(x)$  in each material domain  $\Omega_k$  is scaled by the parameter  $\gamma_k \in \mathbb{R}$ . Now the activation time depends also on the scaling parameters  $\underline{\gamma} \in \mathbb{R}^m$ . One chance to scale the scaling parameters suitably consists in comparing the Eikonal computed activation sequence on the heart surface with the measured ECG on the torso mapped onto this surface. It remains to minimize the functional

$$f(\underline{\gamma}) := \|\varphi^*(x) - \varphi(x, \underline{\gamma})\|_{\ell_2(\omega_h)}^2$$

with respect to  $\underline{\gamma}$ . The vertices in the discretization on the surface of  $\Omega_h$  are denoted by  $\omega_h$ . By minimizing the squared distance between the measured solution  $\varphi^*$  and the Eikonal computed solution  $\varphi(\underline{\gamma}, x)$  we are able to determine the scaling parameters  $\underline{\gamma} \in \mathbb{R}^m$ .

The minimization problem is solved by the quasi-Newton method with BFGS update and adaptive step size control. The gradient  $\nabla_{\underline{\gamma}} f(\underline{\gamma})$  is computed via finite differences or via automatic differentiation using dco++.

We present numerical examples demonstrating the parallel performance of our Eikonal solvers as well as of the optimization. Calculating the gradient via an analytic adjoint-state method is ongoing work.

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## On Block-Krylov Subspace Iterations and the AMP Eigensolver Software

Ming Zhou<sup>1</sup>

This talk is concerned with the efficient computation of the smallest eigenvalues and the corresponding invariant subspace of an FE-discretized self-adjoint and elliptic partial differential operator. We demonstrate an implementation of block-Krylov subspace iterations in the AMP Eigensolver software (<http://www.math.uni-rostock.de/ampe>) which combines adaptive mesh refinement with preconditioned iterations for matrix eigenproblems. Further, we report on recent results [5] on the convergence analysis of block-Krylov subspace iterations. Therein an estimate by Saad [1] is improved by changing the underlying auxiliary vectors, and an estimate by Knyazev [2] is generalized based on our previous results on blockwise gradient iterations [3] and restarted Krylov subspace iterations [4].

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# FFT-Based Proximal Methods for the Computational Homogenization of Inelastic Materials

Matti Schneider<sup>1</sup> Daniel Wicht<sup>2</sup> Thomas Böhlke<sup>3</sup>

In the context of computational homogenization on volumetric image data FFT-based methods have demonstrated their superiority in recent years. From a mathematical perspective, their core is constituted by preconditioning via solving a homogeneous reference problem, and a matrix-free formulation. In this talk we show how (fast) gradient and proximal splitting schemes can be used to solve large scale inelastic homogenization problems for realistic microstructure geometries.

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## Discrete Total Variation with Finite Elements

Roland Herzog<sup>1</sup>

The total-variation (TV) seminorm is ubiquitous as a regularizing functional in image analysis, inverse problems and also optimal control applications. We propose and analyze a discrete analogue of the TV-seminorm for functions belonging to a space of globally discontinuous or continuous (possibly higher order) finite element functions on a geometrically conforming mesh. We show that our discrete TV functional admits a convenient dual representation close to the continuous formulation, which is the basis of many popular solution algorithms.

References:

[1] <http://arxiv.org/abs/1804.07477>

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## Applications of the Discrete Total Variation in Imaging

José Vidal-Núñez<sup>1</sup> Roland Herzog<sup>2</sup> Stephan Schmidt<sup>3</sup> Marc Herrmann<sup>4</sup>  
Gerd Wachsmuth<sup>5</sup>

Taking as starting point the proposed discrete definition of the total-variation (TV) seminorm for Lagrangian finite element functions, we discuss how many popular algorithms for solving TV-L1 and TV-L2 image denoising and related problems can be employed efficiently in this particular setting. Numerical results will be presented and the performance of the algorithms investigated for different polynomial orders.

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# FE Error Estimates for Semilinear Parabolic Control Problems in the Absence of the Tikhonov Term

Arnd Rösch<sup>1</sup> Eduardo Casas<sup>2</sup> Mariano Mateos<sup>3</sup>

We study the FE discretization of a semilinear parabolic optimal control problems without the Tikhonov term. Based on a specific second-order sufficient optimality condition a priori error estimates are derived. The error estimates can be significantly improved for optimal controls with a bang-bang structure. The theoretical results are illustrated by numerical experiments.

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## Optimal Control of a Simplified Signorini Problem

Christof Haubner<sup>1</sup>

The Signorini problem describes the deformation of an elastic body which is pushed against a rigid surface. It can be formulated as an elliptic variational inequality of first kind with unilateral constraints at the boundary. We consider a simplified version governing an optimal control problem. The state is discretized using linear finite elements while a variational discretization is applied to the control. We derive a priori error estimates for the control and state based on strong stationarity and a quadratic growth condition. The convergence rates depend on  $H^1$  and  $L^2$  error estimates of the simplified Signorini problem.

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## Variational Time Discretisations of Higher Order and Higher Regularity

Gunar Matthies<sup>1</sup>   Simon Becher<sup>2</sup>

Starting from the well-known discontinuous Galerkin (dG) and continuous Galerkin-Petrov (cGP) methods we will present a two-parametric family of time discretisation schemes which combine variational and collocation conditions. The first parameter corresponds to the ansatz order while the second parameter is related to the global smoothness of the numerical solution. Hence, higher order schemes with higher order regularity can be obtained by adjusting the family parameters in the right way.

All members of the considered family inherit stability from either dG or cGP. Furthermore, the presented time discretisations can be obtained alternatively by successive post-processing steps starting with dG or cGP, respectively. Furthermore, the considered schemes provide themselves a cheap post-processing which could be used for adaptive time-step control.

Optimal error estimates and some numerical results will be given.

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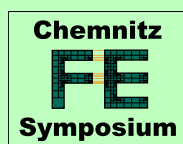
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