# Variational Time Discretization of Higher-Regularity for the Wave Equation

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We present families of variational space-time finite element discretisations for the hyperbolic wave equation, written as the first-order in time system,

$$\partial_t u - v = 0\,, \quad \partial_t v - \nabla \cdot (c \nabla u) = f$$

and equipped with appropriate initial and boundary conditions. This system is studied as a prototype model for elastic waves with applications in, for instance, fluid-structure interaction or non-destructive material inspection.

Firstly, we introduce a family of $C^1$ continuous time discretizations based on a computationally cheap post-processing of continuous in time Petrov–Galerkin approximations; cf. [1, 2]. Optimal order error estimates for the fully discrete scheme are given and illustrated by numerical experiments; cf. [1]. Secondly, a class of schemes that combine collocation and variational equations and ensure higher-order regularity in time is presented; cf. [3]. The convergence properties of its members admitting $C^1$- and $C^2$-regularity in time are analyzed numerically. For a more sophisticated problem of practical interest a comparative study of all schemes is provided.

References:

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