Goal-oriented Adaptivity for Elliptic PDEs with Parametric or Uncertain Inputs

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In this talk, we present a goal-oriented adaptive algorithm for approximating linear quantities of interest derived from solutions to elliptic partial differential equations (PDEs) with parametric or uncertain inputs. Specifically, we consider a class of elliptic PDEs where the underlying differential operator has affine dependence on a countably infinite number of uncertain parameters and employ the stochastic Galerkin finite element method to approximate the solutions to the corresponding primal and dual problems.

Our algorithm follows the standard adaptive loop:

$$\text{SOLVE} \Longrightarrow \text{ESTIMATE} \Longrightarrow \text{MARK} \Longrightarrow \text{REFINE}.$$

Here, the error in the goal functional (e.g., the expectation of the quantity of interest) is *estimated* by the product of computable estimates of the energy errors in Galerkin approximations of the primal and dual solutions. Drawing information from the spatial and parametric contributions to these error estimates, the *marking* is performed by extending the strategy proposed in [3]. Finally, following the methodology developed in [2, 1], a balanced adaptive *refinement* of spatial and parametric components of the approximation space is performed by combining the associated error reduction indicators computed for the primal and dual solutions.

We will discuss the results of numerical experiments that demonstrate the effectiveness of our goal-oriented adaptive strategy for a representative model problem with parametric coefficients and for various quantities of interest (including the approximation of pointwise values).

References:

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