On a Commutator Preconditioner for the Navier-Stokes Equations

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We provide a novel analysis of the PCD (pressure convection-diffusion) preconditioner based on infinite-dimensional interpretation of the underlying operators, which is the basis of operator preconditioning theory. Our results help to answer open questions concerning practical implementation and performance. In particular, we clarify the choice of boundary conditions which is, to our knowledge, a problem missing rigorous analysis, although some heuristic treatment has appeared; see [1, 2]. Moreover, our approach provides a unified theory for both variants (order of the commuted operators) which have thus far only been treated separately in the literature. Last but not least, our infinite-dimensional analysis reveals Fredholm structure in the preconditioner which must be respected in the discretization; failure to do so results in significant deterioration of performance, which has previously been observed but not correctly identified and resolved.

References:

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