Low Rank Tensor Methods for PDE-constrained Optimization with Isogeometric Analysis

Alexandra Bünger[1] Martin Stoll[2]

Isogeometric analysis has become one of the most popular methods for the discretization of partial differential equations motivated by the use of NURBS for geometric representations in industry and science. A crucial challenge lies in the solution of the discretized equations, which we discuss in this talk with a particular focus on PDE-constrained optimization subject to a PDE discretized using IGA.
The discretization results in a system of large mass and stiffness matrices, which are typically very costly to assemble. To reduce the computing time and storage requirements low-rank tensor methods as proposed by Mantzaflaris et al. have become a promising tool. We present a framework for the assembly of these matrices M in low-rank form as the sum of a small number of Kronecker products $M = \sum_{i=1}^{n} \bigotimes_{d=1}^{D} M_i^{(d)}$, where D is the geometry's dimension (2 or 3) and n is determined by the chosen size of the low rank approximation. For assembly of the smaller matrices $M_i^{(d)}$ only univariate integration in the corresponding geometric direction d is required.
The resulting low rank Kronecker product structure of the mass and stiffness matrices can be used to solve a PDE-constrained optimization problem without assembly of the actual system matrices. We present a framework which preserves and exploits the attained Kronecker product format using the *amen block solve* algorithm from the tensor train toolbox in MATLAB to efficiently solve the corresponding KKT system of the optimization problem. We illustrate the performance of the method on various examples.

References:

[1] *https://www.sciencedirect.com/science/article/pii/S0045782516315377*

[2] *https://epubs.siam.org/doi/abs/10.1137/090752286*

[1] TU Chemnitz, Mathematics
`alexandra.buenger@mathematik.tu-chemnitz.de`

[2] TU Chemnitz, Mathematics
`martin.stoll@mathematik.tu-chemnitz.de`