Multigrid Methods for Stochastic Partial Differential Equations

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We study a multigrid method for solving large linear systems of equations with tensor product structure. Such systems are obtained from stochastic finite element discretization of stochastic partial differential equations such as the steady-state diffusion problem with random coefficients. When the variance in the problem is not too large, the solution can be well approximated by a low-rank object. In the proposed multigrid algorithm, the matrix iterates are truncated to low rank to reduce memory requirements and computational effort. The method is proved convergent with an analytic error bound. Numerical experiments show its effectiveness in solving the Galerkin systems compared to the original multigrid solver, especially when the number of degrees of freedom associated with the spatial discretization is large.

References:

[1] *https://www.cs.umd.edu/users/elman/papers/Elman-Su.pdf*

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