# Patient-Specific Cardiac Parametrization from Eikonal Simulations

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Simulations in cardiac electrophysiology use the bidomain equations describing the intercellular and the extracellular electrical potential. Its difference, the trans-membrane potential, is responsible for the excitation of the heart and its steepest gradients form an excitation wavefront propagating in time. This arrival time $\varphi(x)$ of the wavefront at some point $x \in \Omega$ can be approximated by the simpler Eikonal equation. The accuracy of these simulations is limited by unavailable patient specific conductivity data.

The human heart consists of various tissues with different conductivity parameters. We group these tissues into $m$ different classes with its individual scaling parameter $\gamma_k$ yielding to the modified Eikonal equation

$$\sqrt{\left(\nabla\varphi(x,\underline{\gamma})\right)^T \gamma_k \cdot M(x) \nabla\varphi(x,\underline{\gamma})} = 1 \qquad x \in \Omega_k$$

where the velocity information M(x) in each material domain $\Omega_k$ is scaled by the parameter $\gamma_k \in \mathbb{R}$. Now the activation time depends also on the scaling parameters $\underline{\gamma} \in \mathbb{R}^m$.

One chance to scale the scaling parameters suitably consists in comparing the Eikonal computed activation sequence on the heart surface with the measured ECG on the torso mapped onto this surface. It remains to minimize the functional

$$f(\underline{\gamma}) := \parallel \varphi^*(x) - \varphi(x,\underline{\gamma}) \parallel^2_{\ell_2(\omega_h)}$$

with respect to $\underline{\gamma}$. The vertices in the discretization on the surface of $\Omega_h$ are denoted by $\omega_h$. By minimizing the squared distance between the measured solution $\phi^*$ and the Eikonal computed solution $\phi(\underline{\gamma},\underline{x})$ we are able to determine the scaling parameters $\gamma \in \mathbb{R}^m$.

The minimization problem is solved by the quasi-Newton method with BFGS update and adaptive step size control. The gradient $\nabla_\gamma f(\underline{\gamma})$ is computed via finite differences or via automatic differentiation using dco++.

We present numerical examples demonstrating the parallel performance of our Eikonal solvers as well as of the optimization. Calculating the gradient via an analytic adjoint-state method is ongoing work.

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