

Numerical Solution of Isaacs Equations

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Isaacs equations form a family of fully nonlinear PDEs arising naturally from stochastic zero-sum two player games. We consider the problem of the following form:

$$-\delta_t + \inf_{\beta} \sup_{\alpha} (L^{(\alpha,\beta)}v - f^{(\alpha,\beta)}) = 0,$$

where $L^{(\alpha,\beta)}$ are first- or second-order linear operators. One can think of them as a generalisation of Hamilton–Jacobi–Bellman equations from optimal control theory. The aim of the talk is to present a novel finite element method for these Isaacs equations. The main difficulty arises from the non-convex structure of the underlying inf-sup operator. The presented numerical method permits fully implicit, semi-explicit and explicit time discretizations, with the fully implicit one being unconditionally stable. We discuss monotonicity, L-infinity stability and consistency, even for unstructured meshes. Based on this we obtain uniform convergence of numerical solutions to the viscosity solution of Isaacs problem.

References:

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