Efficient Multiphysics Preconditioning for Poroelastic Wave Propagation

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The higher-order mixed-multiphysics space-time finite element discretisation of the dynamic Biot-Allard equations including elastic wave propagation in a fluid-saturated porous media

$$\partial_t((1-\phi)\rho_s \boldsymbol{v} + \phi\rho_f \boldsymbol{q}) - \nabla \cdot \boldsymbol{\sigma}(\boldsymbol{u}, p) = \boldsymbol{f}\,,$$

$\boldsymbol{\sigma}(\boldsymbol{u}, p) = \boldsymbol{C}\boldsymbol{\epsilon}(\boldsymbol{u}) - p\mathbf{1}$, $\boldsymbol{\epsilon}(\boldsymbol{u}) = (\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)/2$, coupled fluid flow

$$\partial_t(c_0 p + \alpha \nabla \cdot \boldsymbol{u}) + \nabla \cdot \boldsymbol{q} = s$$

and $\boldsymbol{q} = -\boldsymbol{K}(\nabla p - \rho_f \boldsymbol{g} - \partial_t \boldsymbol{v})$, equipped with suitable initial and boundary data, results in huge linear systems with a certain block structure. The unknown fields are the displacement $\boldsymbol{u}$, velocity $\boldsymbol{v}$, fluid-pressure p and fluid flux $\boldsymbol{q}$. We solve the arising linear systems in a fully-coupled monolithic way by applying the flexible GMRES iterative linear system solver. In this contribution we present details on our efficient preconditioning strategy consisting of the application of multiple-step fixed-stress iterations in a multigrid in time setting. The performance of the automatically tuned preconditioning strategy and implementation in our distributed-parallel solver suite DTM++ for the deal.II library is analysed carefully with several challenging numerical experiments with relevance to physical problems.

References:

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