High-Order Exactly Divergence-free FEM for Transient Incompressible Flows

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In [1] we presented a unified approach to exactly divergence-free inf-sup stable FEM for the time-dependent incompressible Navier-Stokes problem, covering both H^1- and $H(div)$-conforming methods. Basic features are pressure-robustness, i.e. additional gradient fields $\nabla\psi_h$ in the source term lead to a change$p_h + \psi_h$ of the pressure. This implies that velocity error estimates are not corrupted by large multiples of the best pressure interpolation error. Moreover, the methods are shown to be semi- robust w.r.t. the Reynolds number Re if $u \in L^1(0,T;W^{1,\infty}(\Omega))$, i.e. the error estimates (including the exponential Gronwall factor) do not explicitly on Re.

In this talk, we report on our numerical experience with benchmark problems in 2D and 3D vortex dynamics using high-order FEM including homogeneous, decaying turbulence, see [2, 3]. Moreover, we present some first results on attached boundary layer flows. In particular, we will discuss the question of required numerical diffusion.

References:

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[2] P.W. Schroeder and G. Lube, *Pressure-robust analysis of divergence-free and conforming FEM for evolutionary Navier-Stokes flows*, J. Numer. Math., 25 (2017) 4, pp. 249–276.

[3] P.W. Schroeder and G. Lube, *Divergence-free H(div)-FEM for time-dependent incompressible flows with applications to high Reynolds number vortex dynamics*, J. Sci. Comput. 75 (2018), pp. 830–858.

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