P- and hp-Versions of the Finite Element Method for Poisson's Equation on Polygonal Domains

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We consider boundary value problems for the Poisson equation on polygonal domains with general nonhomogeneous mixed boundary conditions and derive, on the one hand, explicit extraction formulas for the coefficients of the singularities. On the other hand, the formulas are used to construct efficient adaptations for the h-, p- and hp-versions of the finite element method for the numerical treatment. A priori error estimates show that the h-version of the finite element algorithm exhibits the same rate of convergence as it is known for problems with smooth solutions. However, the principal results of the present work are the robust exponential convergence results for the p- and hp-versions of the finite element method on quasiuniform meshes. In fact, it is shown that if the input data (source term and boundary data) are piecewise analytic, then with appropriate choices of conforming finite element subspaces V_N of dimension $N \in \mathbb{N}$, the p- and hp-versions of the finite element algorithms on quasiuniform meshes yield approximate solutions u_N to the exact solution u that satisfy the estimates $\|u - u_N\|_{H^1(\Omega)} \leq C_1 \mathrm{e}^{-b_1 N^{\frac{2}{3}}}$ and $\|u - u_N\|_{H^1(\Omega)} \leq C_2 \mathrm{e}^{-b_2 N^{\frac{1}{3}}}$, respectively. Several numerical experiments are included to illustrate the practical effectiveness of the proposed algorithms. The results show that the theoretical error analyses are attained within the range of engineering computations

References:

[1] B. Nkemzi and S. Tanekou, *Predictor-corrector p- and hp-versions of the finite element method for Poisson's equation in polygonal domains*, Comput. Methods Appl. Mech. Engrg. 333 (2018), pp.74–93

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