Adaptive Stochastic Galerkin FEM driven by Two-Level/Hierarchical Error Estimators for Elliptic Parametric PDEs

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We present an adaptive stochastic Galerkin finite element method for a class of parametric elliptic boundary value problems. The adaptive algorithm is steered by a reliable and efficient *a posteriori* error estimator, which can be decomposed into a two-level spatial estimator and a hierarchical parametric estimator [1]. Following [2, 3], the structure of the estimated error is exploited by the algorithm to perform a balanced adaptive refinement of the spatial and parametric discretizations. The adaptive algorithm is proved to be convergent in the sense that the estimated error converges to zero. Numerical experiments underpin the theoretical findings and show that the proposed adaptive strategy helps to mitigate the 'curse of dimensionality' which usually afflicts the numerical approximation of parametric PDEs.

References:

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