Space-Time Trace-FEM for Solving PDEs on Evolving Surfaces

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We consider the parabolic equation

$$\begin{aligned} \dot{u} + (\mathrm{div}_{\Gamma(t)}\mathbf{w})u - \nu_d \Delta_\Gamma u &= f \quad \text{on } \Gamma(t), \quad t \in (0,T], \\ u(x,0) &= 0 \quad \text{on } \Gamma(0), \end{aligned}$$

posed on a smooth, closed and evolving surface $\Gamma(t)$, which is advected by a given velocity field $\mathbf{w}$. Here, $\dot{u}$ denotes the material derivative. Based on a space-time weak formulation, we present different fully discrete Eulerian finite element methods using DG in time and continuous in space finite elements. The zero level of the space- time linear approximation of the corresponding level set function defines a Lipschitz space- time manifold S_h, which approximates the evolving surface $\Gamma(t)$. Based on volumetric finite elements we use traces on S_h as trial and test surface FE-spaces for discretization.
In a setting with exact and smooth geometry first order error bounds in an energy norm are known to be valid, cf. [1]. In recent work we analyze the effect of the geometry approximation on the discretization error. Furthermore, a space-time surface normal stabilization is introduced that ensures better conditioning properties of the ensuing discretization matrices. In the presentation we explain this space-time Trace-FEM and present results of numerical experiments. Moreover, a few main results of the error analysis are discussed.

References:

[1] M. A. Olshanskii and A. Reusken, *Error analysis of a space-time finite element method for solving PDEs on evolving surfaces*, SIAM Journal on Numerical Analysis, 52(4), 2092–2120, 2014, http://dx.doi.org/10.1137/130936877

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