High-Order Finite Elements, Pressure-Robustness and Incompressible Generalised Beltrami Flows

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We consider FEM for the time-dependent incompressible Navier–Stokes equations:

$$\begin{aligned} \text{Find } (u,p)\colon (0,t_{end}) &\to \mathcal{V} \times \mathcal{Q} \text{ s.t.} \\ \partial_t u - \nu \Delta u + (u \cdot \nabla) u + \nabla p &= f, \\ \nabla \cdot u &= 0. \end{aligned}$$

In particular, different aspects of high-order pressure-robust and non-pressure-robust methods with respect to their performance for generalised Beltrami flows are compared. This comparison is done both theoretically (error estimates) and practically (numerical examples).

For brevity, error estimates are done in the H^1-conforming case whereas numerical experiments are performed using H(div)- and L^2- conforming Discontinuous Galerkin discretisations.

A generalised Beltrami flow is characterised by the property that the convective term has a scalar potential; that is, there is a sufficiently smooth ψ such that $(u \cdot \nabla)u = \nabla \psi$. However, even for flows where this is not strictly true everywhere in the domain, we demonstrate that practically relevant flow *at least locally* can behave like a generalised Beltrami flow.

Due to the additional irrotational inertia force, pressure-robust methods have certain advantages which can lead to drastically less expensive discretisations compared to using non-pressure-robust methods.

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