A Time Dependent Two-Phase Stokes Problem: Well-Posedness and Space-Time FEM Discretization

Igor Voulis[1]   Arnold Reusken[2]

We consider a time dependent Stokes problem with a prescribed, sharp, moving interface. This problem has discontinuous density and viscosity coefficients, a pressure solution that is discontinuous across an evolving interface and an interfacial force (surface tension). This strongly simplified two-phase Stokes equation is a good model problem for the development and analysis of finite element discretization methods for two-phase flow problems. In view of the unfitted finite element methods that are often used for two-phase flow simulations, we are interested in a well-posed variational formulation of this Stokes interface problem in a Euclidean setting.

We discuss the derivation of such a variational formulation in suitable spaces of divergence free functions, as well as the discontinuous-in-time variational formulation involving the pressure variable for the divergence free constraint. This latter formulation is a natural starting point for a space-time finite element discretization with a Discontinuous Galerkin temporal discretization. By combining this DG time-stepping scheme with a spatial XFEM method, we obtain a fully discrete method which is capable of treating discontinuities.

References:

[1] *https://www.springer.com/de/book/9783642196850*
[2] *https://arxiv.org/abs/1803.06339*

[1]RWTH, IGPM, Aachen
   `voulis@igpm.rwth-aachen.de`

[2]RWTH, IGPM, Aachen
   `reuksen@igpm.rwth-aachen.de`