Finite Element Approximation of Second Order PDEs in Non-Divergence Form

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Considered are second-order partial differential equations in a non-divergence form, this is,

$$A\colon \nabla^2 u = f \quad \text{in} \quad \Omega,$$

in some bounded domain $\Omega \subset \mathbb{R}^2$. These kind of equations typically arise as subproblems for the solution of Hamilton-Jacobi-Bellman equations in the context of stochastic optimal control or in the linearization of fully non-linear second-order PDEs. Usually, the coefficients of the matrix A are non-differentiable in these applications and thus, the equations must be discussed in the non-variational form.

We investigate a non-conforming finite element approximation of these problems using higher-order Lagrange-elements for the approximation of u and continuous or discontinuous elements for some discrete Hessian. Of particular interest are a priori and a posteriori error estimates.

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