Error Estimates for a Stabilised Space-Time Finite Element Method for the Wave Equation

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For the discretisation of time-dependent partial differential equations usually explicit or implicit time stepping schemes are used. An alternative approach is the usage of space-time methods, where the space- time domain is discretised and the resulting global linear system is solved at once. In any case CFL conditions play a decisive role for stability.

In this talk the model problem is the scalar wave equation. First, a space-time variational formulation of the wave equation and its discretisation via a space-time approach including a CFL condition are motivated and discussed. Second, to gain a deeper understanding of the CFL condition an ordinary differential equation corresponding to the wave equation is analysed. For this ordinary differential equation an unconditionally stable numerical scheme is introduced. By transferring this idea to the wave equation a stabilised space- time finite element method for the wave equation is presented. For this method stability and error estimates are discussed.

Finally, numerical examples for a one-dimensional and a two-dimensional spatial domain are shown.

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