TECHNISCHE UNIVERSITÄT CHEMNITZ

Fakultät für Mathematik

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Chemnitz FE-Symposium 2019

on Tour in Mülheim an der Ruhr

Chemnitz FE Symposium

Programme

Collection of abstracts

List of participants

Mülheim an der Ruhr, September 09 - 11, 2019

Scientific Topics:

The symposium is devoted to all aspects of finite elements and related methods for solving partial differential equations.

The topics include (but are not limited to):

- Scientific Computing,
- Mechanics/Applications,
- Inverse Problems,
- Optimization with PDEs,
- Uncertainty Quantification.

This year we particularly encourage talks on:

- Least squares FE,
- Computational mechanics,
- Scientific computing.

Invited Speakers:

Fleurianne Bertrand (Humboldt-Universität zu Berlin, Germany)

Philipp Junker (Ruhr-Universität Bochum, Germany,
Bergische Universität Wuppertal, Germany)

Stefan Turek (Universität Dortmund, Germany)

Scientific Committee:

Th. Apel (München), S. Beuchler (Hannover), O. Ernst (Chemnitz), G. Haase (Graz), H. Harbrecht (Basel), R. Herzog (Chemnitz), M. Jung (Dresden), U. Langer (Linz), A. Meyer (Chemnitz), O. Rheinbach (Freiberg), A. Rösch (Duisburg-Essen), O. Steinbach (Graz), M. Stoll (Chemnitz)

Organising Committee:

A. Rösch, N. Obszanski, R. Unger
www.chemnitz-am.de/cfem2019/

Internet Access

The Wolfsburg offers free internet access. Access details can be obtained from the Wolfsburg reception.

Food

The conference fee includes:

- Lunch from 12:30 to 13:30 on all three days of the symposium
- Tea and coffee during breaks
- Conference dinner on Monday.

For participants staying at the Wolfsburg there is a breakfast buffet from 7:45 up to 9:30.

Conference Dinner

The conference dinner will start on Monday at 19:00 at the Wolfsburg.

Excursion

The excursion will take place on Tuesday. We will meet at 14:00 in front of the Wolfsburg. For the "Hike-group" the walking distance to the "Sechs-Seen-Platte" is approximately 6 km. For the way back to the Wolfsburg the participants of the "Hike-group" have the option to hike back or to use the public transport. For both options there are two guides. For the way to the Harbour and back we will use the tram 901. The tram leaves from station "Monning" (10 minutes away from the Wolfsburg).

Programme

Programme for Monday, September 9, 2019

09:00	Opening		Room: 1
	Scientific Computing *Chair*: Arnd Rösch		Room: 1
09:05	**Stefan Turek** . . . 8 Massively parallel & low precision accelerator hardware as trends in HPC - How to use it for large scale simulations allowing high computational, numerical and energy efficiency with application to CFD		
09:55	**Jan Philipp Thiele** . . . 9 A parallel FE solver for the NSE with adaptive grid refinement using PU-DWR		
10:20	*Coffee Break*		-10:50
	Scientific Computing *Chair*: Stefan Turek — Room: 1	**Space Time** *Chair*: Steffen Münzenmaier — Room: 2	
10:50	**Mathias Anselmann** . . . 11 Higher order Galerkin-collocation time discretization for the Navier-Stokes equations with Nitsche's method	**Ulrich Langer** . . . 16 Space-time Finite Element Methods for parabolic initial-boundary value problems with non-smooth solutions	
11:15	**Aurora Ferrja** . . . 12 A parallel procedure of optimization and forecasting techniques applied in energy demand	**Julia Hauser** . . . 17 Space-time Finite Element Methods for Maxwell's equations	
11:40	**Roland Herzog** . . . 13 A discretize-then-optimize approach for PDE-constrained shape optimization problems	**Gunar Matthies** . . . 18 Time step control for variational time discretisations of higher order and higher regularity	
12:05	**Max Winkler** . . . 14 Optimal control of pedestrian dynamics	**Douglas Ramalho Queiroz Pacheco** . 19 Stable space-time Finite Elements for viscous flows	
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Programme for Monday, September 9, 2019 (continued)

	Preconditioning *Chair*: Ulrich Langer — Room: 1	**A Posteriori** *Chair*: Gerhard Starke — Room: 2
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14:10	**Stephan Köhler** . . . 22 FETI-DP solvers and deal.II for problems in dislocation mechanics	**Andreas Rademacher** . . . 26 Dual weighted residual method based error indicators for the local choice of the Finite Element
14:35	**Friederike Röver** . . . 23 A Three-Level extension for the Fast and Robust Overlapping Schwarz (FROSch) preconditioner	**Kemal Suntay** . . . 27 Flux-reconstruction for obstacle problem and a posteriori error estimation
15:00	*Coffee Break*	-15:30
	Fractional and Nonlinear PDEs *Chair*: Gundolf Haase — Room: 1	**Optimal Control** *Chair*: Roland Herzog — Room: 2
15:30	**Nabi Chegini** . . . 29 Wavelet regularization for the Cauchy problem of fractional Helmholtz equation	**Livia Betz** . . . 33 Second-order sufficient conditions for optimal control of non-smooth, semilinear equations
15:55	**Mohadese Ramezani** . . . 30 Application of some high-order numerical formulae in solving time-fractional diffusion equations	**Arnd Rösch** . . . 34 Optimal control of nonmonotone semilinear elliptic equations
16:20	**Reza Mokhtari** . . . 31 Solving generalized KdV-Burgers' equations using a hybridized discontinuous Galerkin method	**Monika Weymuth** . . . 35 A priori error analysis for an optimal control problem governed by a variational inequality of the second kind
16:45	*Break*	-17:00
	Applications *Chair*: Andreas Rademacher — Room: 1	**Optimal Control** *Chair*: Thomas Apel — Room: 2
17:00	**Gonzalo G. de Diego** . . . 37 An analysis of the scaled boundary Finite Element Method	**Jens Baumgartner** . . . 42 Optimal control problems and algebraic flux correction schemes
17:25	**Aurora Ferrja** . . . 38 A modified Arnoldi algorithm to compute transmission eigenvalue of an interior transmission eigenvalue problem	**Fernando Gaspoz** . . . 43 Quasi-best approximation in PDE constraint optimization

Programme for Monday, September 9, 2019 (continued)

17:50	**Lucas Schöbel-Kröhn** . . . . 39 Numerical approximation of a chemotaxis system with logistic growth on networks	**Marita Holtmannspötter** . . . . 44 Numerical analysis for coupled parabolic PDE-ODE systems
18:15	**Hendrik Pasing** . . . . 40 Some approaches on a posteriori error estimation in shape optimization	**Huidong Yang** . . . . 45 Numerical optimal control for parabolic PDEs using a space-time Finite Element Method
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	Computational Mechanics *Chair*: Oliver Rheinbach — Room: 1	
09:00	**Philipp Junker** . . . 47 Novel continuum mechanical material models and their treatment by use of the Finite Element Method	
09:50	**Marco Favino** . . . 48 An accuracy condition for the finite element discretization of Biot's equations	
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	Computational Mechanics *Chair*: Philipp Junker — Room: 1	**Numerical Analysis** *Chair*: Gunar Matthies — Room: 2
10:45	**Bernhard Kober** . . . 50 Reconstruction-based a-posteriori error estimation in stress-based FEM for frictional contact problems	**Philipp Morgenstern** . . . 55 Construction of AS T-splines through local higher-dimensional representations
11:10	**Marcel Moldenhauer** . . . 51 Weakly symmetric stress equilibration and a posteriori error estimation for hyperelasticity	**Johannes Riesselmann** . . . 56 Curl-free Finite Elements for gradient elasticity at finite strains
11:35	**Lisa Julia Nebel** . . . 52 Formation of wrinkles on a coated elastic substrate	**Michael Sievers** . . . 57 Approximation of rate-independent evolution with non-convex energies
12:00	**Korinna Rosin** . . . 53 Adaptive finite cell methods for contact problems	**Timo Sprekeler** . . . 58 Finite Element approximation of elliptic homogenization problems in nondivergence-form
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14:00	*Excursion*	

Programme for Wednesday, September 11, 2019

Least-Squares Finite Element Method
Chair: Sven Beuchler — Room: 1

10:15 *Coffee Break* -10:45

Least-Squares Finite Element Method and High Order
Chair: Fleurianne Bertrand — Room: 1

12:25 Closing — Room: 1

12:30 *Lunch*

Collection of Abstracts

Massively parallel & low precision accelerator hardware as trends in HPC - How to use it for large scale simulations allowing high computational, numerical and energy efficiency with application to CFD

Stefan Turek[1]

The aim of this talk is to present and to discuss how modern, resp., future High Performance Computing (HPC) facilities regarding massively parallel hardware with millions of cores together with very fast, but low precision accelerator hardware can be exploited in numerical simulations so that a very high computational, numerical and hence energy efficiency can be obtained. Here, as prototypical extreme-scale PDE-based applications, we concentrate on nonstationary flow simulations with hundreds of millions or even billions of spatial unknowns in long-time computations with many thousands up to millions of time steps. For the expected huge computational resources in the coming exascale era, such type of spatially discretized problems which typically are treated sequentially, that means one time after the other, are still too small to exploit adequately the huge number of compute nodes, resp., cores so that further parallelism, for instance w.r.t. time, might get necessary.

In this context, we discuss how ”parallel-in-space simultaneous-in-time” Newton-Multigrid approaches can be designed which allow a much higher degree of parallelism. Moreover, to exploit current accelerator hardware in low precision (for instance, GPUs or TPUs), that means mainly working in single precision or even half precision, we discuss the concept of ”prehandling” (in contrast to ”preconditioning”) of the corresponding ill-conditioned systems of equations, for instance arising from Poisson-like problems. Here, we assume a transformation into an equivalent linear system with similar sparsity but with much lower condition numbers so that the use of low-precision hardware might get feasible. In our talk, we provide for both aspects preliminary numerical results as ”proof-of-concept” and discuss the open problems, but also the challenges, particularly for incompressible flow problems.

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A parallel FE solver for the NSE with adaptive grid refinement using PU-DWR

Jan Philipp Thiele[1] Prof. Dr. Thomas Wick[2]

An ongoing trend in Computational Fluid Dynamics the usage of the adaptive finite element method. A common problem arises in the evaluation of specific goal functionals. These functionals could be technical quantities like drag or lift coefficients as well as point-values or (local) averages. The Dual Weighted Residual Method (DWR) described in [3] is successful in solving these problems and has been used in many different CFD applications e.g. [1],[2]

In [4] a different approach to error localization using a partition of unity (PU) for the DWR method was proposed. In this work the PU-DWR is used for goal oriented mesh adaptivity. For this the formulation of the error estimator is derived. Since practical problems - especially in 3D - require a considerable amount of degrees of freedom - regardless of the discretization - a focus in this talk lies in the parallel solution. To validate the method the 2D-1, 3D-1Z and 3D-1Q (steady) test cases as described in [5] are computed. These test cases allow for effectivity analysis on drag, lift and pressure difference goal functionals.

References:

[1] https://arc.aiaa.org/doi/pdf/10.2514/6.2014-0917

[2] https://www.sciencedirect.com/science/article/pii/S0045782512000564

[3] https://www.researchgate.net/profile/Roland_Becker2/publication/2615233_Weighted_A_Posteriori_Error_Control_in_FE_Methods/links/0f31752de623968697000000/Weighted-A-Posteriori-Error-Control-in-FE-Methods.pdf

[4] https://www.sciencedirect.com/science/article/pii/S0377042714004798

[5] https://link.springer.com/chapter/10.1007/978-3-322-89849-4_39

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Notes on session “Scientific Computing”

Higher order Galerkin-collocation time discretization for the Navier-Stokes equations with Nitsche's method

Mathias Anselmann[1] Markus Bause[2]

We present families of families of Galerkin–collocation time discretization schemes for the incompressible Navier–Stokes equations

$$\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} - \nu \Delta \boldsymbol{v} + \nabla p = \boldsymbol{f}\,, \quad \nabla \cdot \boldsymbol{v} = 0\,.$$

The conceptual basis of these schemes is the establishment of a direct connection between the Galerkin method and the classical collocation methods, with the perspective of achieving the accuracy of the former with reduced computational costs in terms of less complex algebraic systems of the latter. Higher regularity in time of the discrete solutions is also ensured; cf. [1,3].

As a further ingredient of our approach, we employ the Nitsche method to impose all types of boundary conditions in a weak form. Thereby, the essential building block for capturing problems of fluid-structure interaction in the future is provided by our approach. For an application of our discretization techniques to elastodynamics and wave problems we refer to [1,2].

References:

[1] M. Anselmann, M. Bause, S. Becher, G. Matthies, *Galerkin–collocation approximation in time for wave equations*, **to appear** (2019), pp. 1–25

[2] M. Bause, U. Köcher, F. A. Radu, F. Schieweck, *Post-processed Galerkin approximation of improved order for wave equations*, Math. Comp., **accepted** (2018), pp. 1–34; arXiv:1803.03005

[3] S. Becher, G. Matthies, D. Wenzel, *Variational methods for stable time discretization of first-order differential equations*, in K. Georgiev, M. Todorov M, I. Georgiev (eds), *Advanced Computing in Industrial Mathematics. BGSIAM*, Springer, Cham, 2018, pp. 63–75

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A parallel procedure of optimization and forecasting techniques applied in energy demand

Aurora Ferrja[1] Eralda Dhamo[2] Besiana Çobani[3] Arbesa Kamberi[4]

The energetic field is one of the most fundamental sector in the development of the economy of a country. To optimize the energy production it is important to obtain an accurate forecast for the demand. The combination of the optimization techniques with time series forecast techniques is studied in other works as well. In this paper we present a parallel algorithm to obtain one day ahead forecasting which consist of combining an evolutionary method and forecasting techniques. The optimization method used is PSO (Particle Swarm Optimization). This method controls a considerable number of restrictions, specifically in our work: the total volume, the maximum and minimum level in HPP, etc, so, it is necessary a parallelization of the work. On the other hand, the forecasting procedure utilize some exogenous variables in a dynamic way to obtain the most accurate forecast which serves as an input to PSO. To achieve the required result in a faster execution time a parallel implementation procedure of these two techniques is used. Daily data of energy production are analyzed and forecast.

References:

[1] Makridakis S, Spiliotis E, Assimakopoulos V (2018) Statistical and Machine Learning forecasting methods: Concerns and ways forward. PLoS ONE 13(3): e0194889. https://doi.org/10.1371/journal.pone.0194889

[2] David A. Harpman, 2009 Scoping report-new algorithms for hydropower optimization U.S Bureau of Reclamation's Science and Technology

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A discretize-then-optimize approach for PDE-constrained shape optimization problems

Roland Herzog[1] Estefania Loayza[2]

In the traditional approach to algorithmic PDE-constrained shape optimization, one derives a formula for the shape derivative (involving an adjoint PDE) either in volume or boundary formulation, then converts it into a shape gradient (vector field), which drives the transformation of the current into an updated domain. When implementing a discrete version of this procedure, the mesh quality often degrades.

In this talk, we propose a fully discrete approach to PDE-constrained shape optimization. We define a finite dimensional manifold of discrete shapes represented by computational meshes and perform mesh updates along the geodesics of a complete Riemannian metric. This procedure avoids mesh degradation by design. Numerical examples will be included.

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Optimal control of pedestrian dynamics

Max Winkler[1]

This talk is devoted to optimal control problems for the Hughes model which is a mathematical description of pedestrian dynamics. As an application, we consider the optimal evacuation of a crowd in e. g. a burning building. The typical observation is that all people run to the closest exit and depending on the initial distribution of the crowd some exits slightly further away are not minded. Thus, we modified the model and introduced so-called agents which may also attract the crowd and the aim is to control the movement of these agents such that the evacuation is optimized by evenly spreading the crowd to all exits.

The model is based on a coupled system of a transport equation for the crowd, an Eikonal equation for the potential and an ODE system for the agent dynamics. The problem is discretized with a discontinuous Galerkin scheme. Moreover, we investigate first-order necessary optimality conditions and gradient based optimization methods for the optimal control problem.

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Notes on session “Scientific Computing”

Space-time Finite Element Methods for parabolic initial-boundary value problems with non-smooth solutions

Ulrich Langer[1] Andreas Schafelner[2]

We consider locally stabilized, conforming finite element schemes on completely unstructured simplicial space-time meshes for the numerical solution of parabolic initial-boundary value problems with variable, possibly discontinuous in space and time coefficients. Discontinuous coefficients, non-smooth boundaries, changing boundary conditions, non-smooth or incompatible initial conditions, and non-smooth right-hand sides can lead to non-smooth solutions. For instance, in electromagnetics, permanent magnets cause line-delta-distributions in the source term in 2d quasi-magnetostatic simulations of electrical machines.

We present new a priori and a posteriori error estimates for low-regularity solutions. In order to avoid reduced convergence rates appearing in the case of uniform mesh refinement, we also consider adaptive refinement procedures based on residual a posteriori error indicators and functional a posteriori error estimators. The latter provides guaranteed upper bounds on the error. The huge system of space-time finite element equations is then solved by means of GMRES preconditioned by space-time algebraic multigrid. In particular, in the 4d space-time case that is 3d in space, simultaneous space-time parallelization can considerably reduce the computational time. We present and discuss numerical results for several examples possessing different regularity features. The implementation is performed within MFEM.

The authors would like to thank the Austrian Science Fund (FWF) for the financial support under the grant DK W1214-04.

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Space-time Finite Element Methods for Maxwell's equations

Julia Hauser[1] Olaf Steinbach[2]

We consider Maxwell's equations in a space-time setting and the corresponding variational formulations. In particular we take a look at the vectorial wave equation for the electric field E including the spatial curl operator. By applying integration by parts in both time and space we derive a Galerkin-Petrov formulation for which we will discuss unique solvability under different assumptions on the given data. Although the numerical discretization in a 4D space-time setting seems to be ambitious at a first glance, it allows for an adaptive resolution simultaneously in time and space and for a parallel implementation. In the end we will consider examples and open problems.

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Time step control for variational time discretisations of higher order and higher regularity

Gunar Matthies[1] Simon Becher[2]

As generalisation of the well-known discontinuous Galerkin (dG) and continuous Galerkin-Petrov (cGP) methods, a two-parametric family of variational time discretisations has been proposed recently. The two family parameters allow to control the ansatz order and global smoothness of the solution. Hence, higher order schemes with higher order regularity can be obtained by adjusting the family parameters in the right way.

Many variational time discretisation methods provide a cheap post-processing leading to a solution that converges in integral-based norm of one order higher than those of the original methods. Hence, the difference between the original and the post-processed solution can be used within an adaptive time-step control. We will discuss the behaviour in dependence of the two family parameters of the variational time discretisation schemes.

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Stable space-time Finite Elements for viscous flows

Douglas Ramalho Queiroz Pacheco[1] Olaf Steinbach[2]

Space-time finite element formulations differ from conventional methods by treating time and space in a similar manner. This means that time is considered as an additional dimension, so that the whole space-time domain is discretized with finite elements. Such formulations have proven attractive for the solution of various problems, for instance, by offering a natural framework to tackle flow problems with time-dependent domains. In the present work, the well-known concept of the Taylor-Hood element is extended for the Galerkin space-time variational formulation of the Stokes system. Classical spatial Taylor-Hood elements are constructed by interpolating flow velocities with a one order higher polynomial degree than the pressure. This guarantees the fulfillment of a discrete inf-sup condition, thereby yielding a stable method. As our main contribution, we construct space-time Taylor-Hood elements by using second-order interpolation for the velocity and first-order interpolation for the pressure – with the same degree for space and time. This yields an unconditionally stable and optimally convergent method. We further consider two types of elements, namely, prismatic (simplicial in space, tensor product in time) and tetrahedral (fully simplicial). A comparative study between these two different Taylor-Hood-type elements is carried out, and numerical simulations reveal the potential of the method.

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Notes on session “Space Time”

On PCD preconditioner for Navier-Stokes equations

Jan Blechta[1]

We provide a novel analysis for the pressure convection-diffusion (PCD) preconditioner for the incompressible Navier-Stokes equations. We first develop a theory for the preconditioner considered as an operator in infinite-dimensional spaces. We then provide a methodology for constructing discrete PCD operators for a broad class of pressure discretizations. The principal contribution of the work is that a clear and pronounced methodology for dealing with the artificial boundary conditions is given, including the inflow-outflow case, which has not been adequately addressed in the existing literature. In particular, new forms of discrete PCD are derived, which are, unlike the previously published variants, proven to be invertible and robust in data.

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FETI-DP solvers and deal.II for problems in dislocation mechanics

Stephan Köhler[1] Oliver Rheinbach[2] Stefan Sandfeld[3] Dominik Steinberger[4]

FETI-DP (Finite Element Tearing and Interconnecting Dual-Primal) solvers and the deal.II adaptive finite element library are combined to solve dislocation eigenstrain problems in micromechanics. Computational results using adaptive finite elements with millions of unknowns are presented.

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A Three-Level extension for the Fast and Robust Overlapping Schwarz (FROSch) preconditioner

Friederike Röver[1] Alexander Heinlein[2] Axel Klawonn[3] Oliver Rheinbach[4]

The Fast and Robust Overlapping Schwarz (FROSch) framework in the Trilinos software library contains a parallel implementation of the two-level GDSW overlapping Schwarz preconditioner using an energy-minimizing coarse space. It can be constructed algebraically from the assembled matrix. To improve the parallel scalability of the two-level method, a three-level extension has been introduced, recently. Numerical results in two and three dimensions are presented. A further improvement of the scalability can be obtained by an approach using a reduced coarse space. Results for the three- and two-level method applying the reduced coarse space are also presented. Regarding the size of the coarse problem, the new methods can be expected to scale when the classical method will be out of memory.

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Notes on session “Preconditioning”

A general framework for applying the DWR method on variational inequalities

Ullrich Heupel[1] Andreas Rademacher[2]

In this talk, we consider variational inequalities of first and second kind including a smooth nonlinear differential operator. We rewrite the variational inequality with the help of nonlinear complementarity (NCP) functions as a nonlinear problem. However, due to the nonsmoothness of the NCP functions, the resulting problem is not smooth. Furthermore we have potentially to assume additional properties of the analytic solution to do the reformulation. We want to apply the dual weighted residual (DWR) method to estimate the discretization error in a user defined quantity of interest. Due to the nonsmoothness of our formulation, we cannot directly apply the classic DWR approach, where the problem is linearized on the basis of the directional derivative of the underlying semi linearform. Instead, we use the active sets provided by the NCP functions to do the linearization w.r.t. them, while the differential operator is treated in the classic way. The arising dual problem is closely connected to the linear system of equations, which has to be solved in the last step of a semi smooth Newton method applied to the original problem. We derive an error identity, which consists in the primal residual, the dual residual and a remainder term. The remainder term is neglectable. The primal residual and the dual residual are weighted by the dual and the primal discretization error, respectively. Thus, their evaluation has to be approximated numerically. Finally, the error estimate is localized to the mesh cells by a filtering approach to utilize it in an adaptive strategy. As an example for the application of this framework we discuss Signorini's problem with friction discretized by a mixed method. Finally, we give an outlook on thermoplastic contact problems.

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Dual weighted residual method based error indicators for the local choice of the Finite Element

Andreas Rademacher[1] Dustin Kumor[2]

The contribution at hand deals with locking effects within the simulation of problems with local regions of nearly incompressible material behaviour and the possibility to cope with these difficulties by an adaptive choice of the finite element. Here, we consider the problem of linear elasticity, which is discretised with standard bilinear finite elements. It is a well known fact, that this kind of discretisation leads to locking phenomena in the case that Poisson's ratio is close to 0.5. The modification of the continuous bilinear form on the discrete level is one possibility to overcome the drawback of the initial formulation. We discretise the bilinear form by using a discrete divergence operator leading to the discretised bilinear form, which is realised by applying a one point Gaussian quadrature rule for the corresponding scalar product. This approach is known as selective reduced integration.

The discretisation error with respect to a user defined quantity of interest can be estimated applying the dual weighted residual (DWR) method. Since the discrete solution is computed by using a modified discrete bilinear form, the standard approach of the DWR method is not applicable and needs to be modified. We derive an error identity involving some additional terms due to the different bilinear forms and discuss the numerical approximation of it, which has to be handled with special care.

A further aim is to consider the adaptive local choice of the bilinear form to compute the element matrix. The adaptive choice of the bilinear form on element level describes a problem with model adaptive character. We use the DWR method to estimate the difference between the two arising discretisation errors. Finally, numerical results substantiate the accuracy of the presented error estimator and the efficiency of the adaptive algorithms.

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Flux-reconstruction for obstacle problem and a posteriori error estimation

Kemal Suntay[1] Gerhard Starke[2]

In this talk we consider the elliptic obstacle problem for a membrane. We solve this problem with P2 nonconforming finite elements the so-called Fortin-Soulie elements, see [1]. In [2] Fortin-Soulie elements were used successfully to reconstruct a H(div)-conforming flux with application to a-posteriori error estimation. We reconstruct a flux for the obstacle problem using these P2 noncorming finite element and use this flux for a a- posteriori error estimation based on the dual formulation of the obstacle problem. The basic idea of the estimation can be traced back to Prager and Synge [3]. Finally numerical results will be presented.

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Notes on session “A Posteriori”

Wavelet regularization for the Cauchy problem of fractional Helmholtz equation

Nabi Chegini[1] Hussein Lotfinia[2]

This talk is devoted to the solution of the Cauchy problem related to fractional Helmholtz equation. Since this problem is ill-posed, a regularization method is needed to overcome the ill-posedness. For this task, we combine two powerful tools that are Fourier transform and wavelet filtering. To regularize this problem with noisy measured data, we use Meyer wavelet with a suitable level of approximation, as a low-pass filter to remove high frequencies which cause the ill-posedness. Then we transform the problem to frequency space to solve it by Fourier transform. In practical examples we use fast Fourier transform that makes the method very fast and effective. An analytic discussion, indicates the convergence of the method rather than a numerical implementation is proposed in details.

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Application of some high-order numerical formulae in solving time-fractional diffusion equations

Mohadese Ramezani[1] Reza Mokhtari[2] Gundolf Haase[3]

A time-fractional diffusion equation with the Caputo fractional derivative is considered and solved by using a numerical scheme based on finite element method and/or finite difference method in space. For the Caputo fractional derivative of order α, $0 < \alpha < 1$, some formulae such as L1, L1-2 and L1-2-3 had been constructed previously by using the piecewise Lagrange interpolation and some new formulae based on B-spline interpolations had been introduced and called S1, S2 and S3 with $2-\alpha$, $3-\alpha$ and $4-\alpha$ order of convergence, respectively. The advantage of the B-spline based formulae lies in the fact that the form of these new formulae is as simple as L1 formula with fixed accuracy in the whole interval of integration while the previous formulae such as L1-2 have lower accuracy at the beginning of the interval. Finally, some numerical examples are tested in order to demonstrate the applicability and accuracy of the new formulae.

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Solving generalized KdV-Burgers' equations using a hybridized discontinuous Galerkin method

Reza Mokhtari[1] Shima Baharlouei[2]

In this talk, we aim to proceed to construct and analyze a hybridized discontinuous Galerkin (HDG) method for solving generalized KdV-Burgers' (KdVB) equations. Following KdVB equation is considered

$$\mathfrak{u}_t + f(\mathfrak{u})_x - (a(\mathfrak{u})\mathfrak{u}_x)_x + (r'(\mathfrak{u})g(r(\mathfrak{u})_x)_x)_x = 0, \qquad x \in \Omega \subset \mathbb{R}, \qquad t \in (0,T] \tag{1}$$

where f, a, r and g are some given functions. For the sake of simplicity, we consider a special case of equation (1) as follows

$$\mathfrak{u}_t + f(\mathfrak{u})_x - \alpha_1 \mathfrak{u}_{xx} + \alpha_2 \mathfrak{u}_{xxx} = 0, \qquad x \in \Omega \subset \mathbb{R}, \qquad t \in (0,T] \tag{2}$$

in which $\alpha_1 > 0$, $\alpha_2 \in \mathbb{R}$, and f is a suitable function which is in most problems as $f(\mathfrak{u}) = \frac{\alpha_0}{n+1}\mathfrak{u}^{n+1}$. We focus on equation (2) because it is more applicable in engineering and physics in particular it has many applications in plasmas and fluids. On the other hand, the HDG method is one of the outstanding and successful methods for solving nonlinear evolution equations. Hence, we exploit an HDG discretization in space and an Euler method in time. A theorem related to the stability of the proposed method is proved. More precisely, we prove that if equation (2) is equipped with periodic or appropriate homogeneous Dirichlet boundary conditions then the proposed HDG method is stable subject to proper choice of stabilization parameters. We examine some different examples to observe optimal convergence in both approximate solution and its derivative. Propagation and interaction of some solitons such as the bell-type and/or kink-type and the evolution of the shock-wave solutions are tested to demonstrate the effectiveness and applicability of the proposed method.

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Notes on session "Fractional and Nonlinear PDEs"

Second-order sufficient conditions for optimal control of non-smooth, semilinear equations

Livia Betz[1]

This talk is concerned with an optimal control problem governed by a non-smooth, semilinear elliptic PDE. The nonlinearity in the state equation is only directionally differentiable, locally Lipschitz continuous, and is allowed to have infinitely many non-differentiable points. By employing its limited properties, Bouligand-differentiability of the control-to-state map is shown. This enables us to establish second-order sufficient optimality conditions.

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Optimal control of nonmonotone semilinear elliptic equations

Arnd Rösch[1] Eduardo Casas[2] Mariano Mateos[3]

In this talk we study optimal control problems governed by a semilinear elliptic equation. The equation is nonmonotone due to the presence of a convection term, despite the monotonicity of the nonlinear term. The resulting operator is neither monotone nor coercive. We present several theoretical and numerical results.

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A priori error analysis for an optimal control problem governed by a variational inequality of the second kind

Monika Weymuth[1] Christian Meyer[2]

We consider an optimal control problem governed by an elliptic variational inequality of the second kind. The problem is discretized by linear finite elements for the state and a variational discrete approach for the control. We present nearly optimal a priori error estimates, i.e. we prove second order convergence (up to logarithmic terms) for the state and first order convergence (up to logarithmic terms) for the control. The key tools for the proof are strong stationarity conditions and a quadratic growth condition. The derivation of these strong stationarity conditions is based on differentiability properties of the control-to-state operator and needs only mild assumptions on the active set.

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Notes on session “Optimal Control”

An analysis of the scaled boundary Finite Element Method

Gonzalo G. de Diego[1] Fleurianne Bertrand[2]

The scaled boundary finite element method (SBFEM) is a relatively novel approach to the approximation of the solution of a PDE introduced in the engineering community. Given a star-shaped domain, the scaled boundary transformation is performed such that the domain can be represented in terms of a radial coordinate and circumferential coordinates. By discretising along the circumferential coordinates only, the PDE can be rewritten as an ODE for which an analytical solution can be found under certain conditions.

In this presentation, a theoretical framework for the analysis of SBFEM is proposed. In particular, the Poisson equation is considered on a “Pacman” domain and an adequate subspace of $H^1(\Omega)$ is defined in which to seek solutions constructed with SBFEM. The well-posedness of SBFEM, a priori error estimates and several numerical examples are discussed.

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A modified Arnoldi algorithm to compute transmission eigenvalue of an interior transmission eigenvalue problem

Aurora Ferrja[1] Besiana Çobani[2] Eralda Dhamo[3] Fatmir Hoxha[4]

The inverse scattering plays a crucial role in the modern field of partial differential equations. It is a special field of interest for many mathematicians who deal with partial differential equations theory and it is in continuous process. The nature of the inverse problem is generally ill-posed, more specifically the third condition fails (the solution does not depend on the initial data), so it is a real challenge to find a solution to the given problem. In this paper we deal with an interior transmission problem which is a boundary value problem compounded of two partial differential equations of second order defined in a bounded domain that correspond to the scatterer. Its homogeneous version is referred to as the transmission eigenvalue problem, which is nonlinear and non self-adjoint eigenvalue problem. After proving the discreetness and the existence of the following problem

$$
\begin{aligned}
&\Delta w + k^2 \ nw = 0 \ \text{ in } \Omega \\
&\Delta v + k^2 \ v = 0 \ \text{ in } \Omega \\
&w - v = -\eta \ \partial v / \partial \vartheta \ \text{ in } \partial\Omega \\
&\partial w / \partial \vartheta - \partial v / \partial \vartheta = 0 \ \text{ in } \partial\Omega
\end{aligned}
$$

we focus in a numerical method that gives the first value of k. (the first eigenvalue) . More precisely, we use the finite element method. We transform the transmission problem into a weak problem. Then we use standard piecewise linear finite elements to discretize this problem. We need only a few lowest real values of transmission eigenvalue in inverse scattering theory, so a modified algorithm using Arnoldi method is used to compute these eigenvalues. We use Matlab for the implementation because it is more convenient to use the finite element method.

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Numerical approximation of a chemotaxis system with logistic growth on networks

Lucas Schöbel-Kröhn[1]

In this talk we consider a chemotaxis system with a logistic growth term on a network structure. It is shown that there exists a unique weak solution which is bounded polynomially in time and, as a consequence, exists globally. Moreover, a finite element method which is modified by mass lumping and upwinding techniques to ensure conservation of mass and positivity on the discrete level is developed. Convergence of the method is proved under general assumptions on the data and optimal convergence rates are obtained if the solution is sufficiently regular. The theoretical findings are illustrated by some numerical examples. This work generalizes the results obtained in the paper "Chemotaxis on networks: Analysis and numerical approximation" by H. Egger and the author by considering additionally a logistic growth term.

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Some approaches on a posteriori error estimation in shape optimization

Hendrik Pasing[1]

After a brief introduction to shape optimization we will discuss approaches on a posteriori error estimation, including a posteriori error estimation of the compliance and shape gradient approximation. In addition to the aforementioned approaches we will present associated open or pending questions. In general we will assume models of elastic structures if necessary.

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Notes on session “Applications”

Optimal control problems and algebraic flux correction schemes

Jens Baumgartner[1] Arnd Rösch[2]

Solutions of convection-diffusion-reaction equations may possess layers, i.e., narrow regions where the solution has a large gradient (in particular for convection dominated equations). Standard finite element methods lead to discrete solutions which are polluted by spurious oscillations. The main motivation for the construction of the so-called algebraic flux correction (AFC) schemes is the satisfaction of the DMP to avoid spurious oscillations in the discrete solutions. We apply an AFC scheme to an optimal control problem governed by a convection-diffusion-reaction equation. Due to the fact that the AFC schemes are nonlinear and usually non-differentiable the approaches "optimize-then-discretize" and "discretize-then-optimize" do not commute. We use the "optimize-then-discretize" approach, i.e., we discretize the state equation and besides the adjoint equation with the AFC method.

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Quasi-best approximation in PDE constraint optimization

Fernando Gaspoz[1] Christian Kreuzer[2] Andreas Veeser[3] Winnifried Wollner[4]

We consider finite element solutions to quadratic optimization problems, where the state depends on the control via a well-posed linear partial differential equation. Exploiting the structure of a suitably reduced optimality system, we prove that the combined error in the state and adjoint state of the variational discretization on FEM spaces is bounded by the best approximation error in the underlying discrete spaces. The constant in this bound depends on the inverse square-root of the Tikhonov regularization parameter. Furthermore, if the operators of control-action and observation are compact, this quasi-best-approximation constant becomes independent of the Tikhonov parameter as the meshsize tends to 0 and we give quantitative relationships between meshsize and Tikhonov parameter ensuring this independence. We also derive generalizations of these results when the control variable is discretized or when it is taken from a convex set.

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Numerical analysis for coupled parabolic PDE-ODE systems

Marita Holtmannspötter[1]

In this talk we investigate a priori error estimates for the space-time Galerkin finite element discretization of an optimal control problem governed by a simplified damage model. The model equations are of a special structure as the state equation consists of a coupled parabolic PDE-ODE system. Among other things, challenges for the derivation of error estimates arise from low regularity properties of solutions provided by this system. The state equation is discretized by a piecewise constant discontinuous Galerkin method in time and usual conforming linear finite elements in space. We provide error estimates both for the discretization of the state equation as well as for the optimal control. Numerical experiments are added to illustrate the proven rates of convergence.

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Numerical optimal control for parabolic PDEs using a space-time Finite Element Method

Huidong Yang[1] Ulrich Langer[2] Olaf Steinbach[3] Fredi Tröltzsch[4]

In this talk, we will present some numerical methods for optimal control of parabolic PDEs. In particular, we aim to minimize certain objective functionals subject to linear/nonlinear parabolic PDEs, under proper constraints on the control variables. We use a space-time finite element method, based on a Galerkin–Petrov variational formulation employing piecewise linear finite elements simultaneously in space and time, to discretize the optimality system, which includes both the state and adjoint state equations. In contrast to the main drawback of conventional time stepping methods, such an approach provides a flexibility to perform local refinements in space and time simultaneously for the coupled optimality system, in which time is considered as another spatial coordinate. On the other hand, it often becomes a bottleneck to efficiently solve the large scale linear/linearized system of equations arising from the space-time finite element discretization, for which we have considered an algebraic multigrid preconditioned GMRES method.

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Notes on session “Optimal Control”

Novel continuum mechanical material models and their treatment by use of the Finite Element Method

Philipp Junker[1]

The system of principal physical equations is not closed for until additional equations are formulated which take the specific material behavior into account. Usually, additional, so-called internal variables are introduced that describe the time dependent microstructure and whose evolution is modeled in terms of ordinary differential equations (ODEs) or partial differential equations (PDEs), respectively. This general approach of material modeling has successfully been applied to various problems of engineering interest.

This talk aims at presenting a short overview on how the finite element method can be used to find approximate solutions to the mentioned system of coupled differential equations by focusing on novel approaches for two different problems: visco-elastic materials with stochastic properties and thermodynamic topology optimization including material nonlinearities.

After presenting a variational strategy for general material modeling, the problem of visco-elastic materials with stochastic properties is discussed. In this case, the material behavior is described by an ODE with stochastic coefficients. The recently developed time-separated stochastic mechanics allows splitting the time-independent, stochastic terms from the time-dependent deterministic terms. Thereby, both the expectation and the variance for the mechanical stresses can be computed instantly. In the end, a finite element implementation needs only negligibly more computation time than one stochastic realization hundreds of which are needed for referential Monte Carlo simulations to converge.

In the second part of the talk, the thermodynamic topology optimization is presented which makes use of a gradient-enhanced energy functional for regularization. The resultant Helmholtz equation is solved in an efficient manner which avoids increasing the number of nodal unknowns. Since the PDE for the density distribution of the topology is derived from thermodynamic concepts, it can be equipped by material models that account for material nonlinearities such as anisotropy and tension/compression asymmetry.

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An accuracy condition for the finite element discretization of Biot's equations

Marco Favino[1]

Biot's equations are a system of time-dependent partial differential equations, which describe the mechanical behaviour of fluid-saturated porous media [1]. The unknowns of such a system are the solid displacement and in the pore pressure.

Finite element discretizations of Biot's equations may exhibit unphysical oscillations in the solid displacement and in the pore pressure, which tend to increase when the time step is reduced [2]. In the one-dimensional case, a lower limit for the time step has been derived, in terms of the mesh size and the mechanical properties.

Differently from previous works, e.g. [2, 3], we identify the unphysical oscillations as a violation of a generalized discrete maximum principle and, in this way, we extend the lower limit for the time-step to the two- and three-dimensional cases [4].

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Notes on session "Computational Mechanics"

Reconstruction-based a-posteriori error estimation in stress-based FEM for frictional contact problems

Bernhard Kober[1] Gerhard Starke[2] Gabriele Rovi[3] Rolf Krause[4]

The use of stress-based finite element methods for the treatment of contact problems admits locking free performance in the incompressible limit as well as direct access to the surface forces at the contact zone. Consequently we are studying the application of the stress-based FEM described in [1] featuring next-to-lowest order Raviart-Thomas-Elements to the Signorini contact problem with Coloumb friction using a dual variational formulation similar to the one studied in [2].

Since frictional contact problems tend to feature singularities, adaptive refinement strategies are to be considered and reliable a-posteriori error estimation is needed. We therefore extend the a-posteriori error estimator in [5] to frictional contact and reconstruct a H^1-conforming displacement following the ideas in [3] and [4]. We prove reliability of our error estimator under similar assumptions as those made in [6] for uniqueness and test its efficiency by numerical experiments in two and three dimensions.

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Weakly symmetric stress equilibration and a posteriori error estimation for hyperelasticity

Marcel Moldenhauer[1] Fleurianne Bertrand[2] Gerhard Starke[3]

By extending the techniques in [1] for the linear elastic case, an algorithm to reconstruct a H(div)-conforming weakly symmetric stress tensor for the non-linear hyperelastic case is presented. This work builds upon [2] where a local weakly symmetric stress reconstruction is derived for arbitrary conforming finite elements in linear elasticity. The reconstructed stress tensor is used as an a posteriori error estimator. Numerical results for the incompressible hyperelastic case are presented.

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Formation of wrinkles on a coated elastic substrate

Lisa Julia Nebel[1]

We consider a thin elastic substrate coated with a hard film that forms wrinkles when stretched or sheared. The elastic substrate is modeled using a hyperelastic, homogeneous and isotropic material. The film is modeled using a Cosserat shell as proposed by Neff. This involves a microrotation field of orthonormal director triads on the coated part of the substrate boundary, which will be denoted by Ω_ξ.
The material behavior can be described by the sum of two energies:

$$J(\varphi, R) = \int_\Omega W_e(\nabla\varphi)\, dV + \int_{\Omega_\xi} W_c(\varphi_{|\Omega_\xi}, R)\, dS$$

where $W_e : GL^+(3) \to \mathbb{R}$ is the elastic energy density and $W_c : GL^+(2) \times SO(3) \to \mathbb{R}$ is the Cosserat shell energy density depending on the surface deformation function φ and the microrotation field R.
We discretize the problem using finite elements for the substrate displacement and geodesic finite elements for the microrotation field. Geodesic finite elements are a generalization of standard finite elements to spaces of functions mapping into a Riemannian manifold.
We obtain an algebraic optimization problem on a finite-dimensional Riemannian manifold. To solve this problem we propose a distributed Riemannian trust-region solver with a monotone multigrid method for the constraint inner problems. Numerical experiments show that we can efficiently reproduce wrinkling patterns of the coupled system.

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Adaptive finite cell methods for contact problems

Korinna Rosin[1] Andreas Rademacher[2]

In this talk, we extend our work on the adaptive optimal control of contact problems [1] onto the discretization by means of the finite cell method (FCM), which is a particular combination of the fictitious domain concept and finite element method. We present only the results for the simulation of the contact problems, in order to focus on the FCM. A more complete work on this topic is given in [3], which also covers the optimal control of contact problems. The underlying contact problems lead to variational inequalities, which have non-regular solution operators, yet we want to apply Newton's method. Therefore, we regularize the non-smooth problems by penalization and afterwards discretize them using the FCM. The latter method allows for a different domain on the discrete and the continuous level. This is in particular useful, if the original domain has a difficult shape or changes during a time-dependent process. We briefly discuss the a-priori FCM-error for the resulting semi-linear problem.

As both modifications (penalization and FCM) introduce great challenges to the conditioning, an adaptive algorithm is required to manage these numerical difficulties as well as the discretization error. Applying the dual weighted residual method, an error identity for the error measured by a goal functional is derived. This identity respects the different error sources. Extending the ideas presented in [1] and [2], we finally deduce computable error estimators concerning the different sources. We present the actual estimators as well as numerical results substantiating the accuracy of the a-posteriori error estimators and the efficiency of the adaptive algorithm.

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Notes on session "Computational Mechanics"

Construction of AS T-splines through local higher-dimensional representations

Philipp Morgenstern[1] Roland Maier[2] Thomas Takacs[3]

In 2003, T-splines were introduced in the context of CAD as a new realization for B-splines on irregular meshes that does not require the bookkeeping of a hierarchical basis, but nevertheless allows for local mesh refinement in order to control small-scale geometry features. Shortly after, IGA was introduced, and T-splines were applied with promising results, however an appropriate local refinement strategy needed to be investigated. The main issues have been solved for 2D meshes until 2013, but spline-based discretizations and appropriate local refinement strategies for unstructured 2D and 3D meshes are still subject of ongoing research.

We propose a construction of analysis-suitable T-splines for unstructured two-dimensional T-meshes. This construction makes locally use of higher-dimensional meshes of which the given unstructured mesh is a lower-dimensional projection. According to this construction, we use key ideas from the local refinement of higher-dimensional structured meshes to develop a local refinement strategy for the 2D unstructured case. Finally, we sketch ideas and ongoing work towards a theoretical rate-optimality of the h-adaptive IGA for a second-order elliptic PDE.

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Curl-free Finite Elements for gradient elasticity at finite strains

Johannes Riesselmann[1] Jonas Ketteler[2] Mira Schedensack[3] Daniel Balzani[4]

Gradient elasticity formulations can be used to model specialized materials, whose elastic behavior depends on size relations between the microstructure and specimen size. Through enrichment of the internal elastic energy by a higher order gradient term, which contains additional constitutive parameters, these so called size effects can be captured. Moreover, singularities at e.g. sharp corners of the modeled specimen, which would appear in classical elasticity formulations are avoided and corresponding numerical simulations have the advantage of remaining mesh objective. However, a straightforward discretization requires C1-continuity, for which retaining compatibility with standard software and meshing arbitrary structures are known difficulties. Another approach is to use mixed formulations instead. They have the advantage of a relaxation of continuity requirement to C0 and making standard finite element discretizations possible. So far, a common approach is to make both displacements, displacement gradients and additional Lagrange multipliers seperate discretization variables leading to finite elements with a relatively high number of degrees of freedom. The present contribution investigates a stable approach in which the displacements are decoupled from the formulation shrinking the size of the problem. Moreover, further cost reduction through reduced numerical approaches eliminating the lagrange multiplier variable are investigated and comparative studies are presented.

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Approximation of rate-independent evolution with non-convex energies

Michael Sievers[1] Christian Meyer[2]

Rate-independent systems governed by non-convex energies provide a several mathematical challenges. Since solutions may in general show discontinuities in time, the design of a suitable, mathematically rigorous notion of solution is all but clear and several different solution concepts exist, such as weak, differential, and global energetic solutions. In the recent past a new promising solution concept was developed, the so-called parametrized solution. The principal idea is to introduce an artificial time, in which the solution is continuous, and to interpret the physical time as a function of the artificial time. A numerical scheme that allows to approximate this class of solutions is the so-called local time-incremental minimization scheme. We investigate this scheme (combined with a standard finite element discretization in space) in detail, provide convergence results in the general case, and prove convergence rates for problems with (locally) convex energies. Numerical tests confirm our theoretical findings.

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Finite Element approximation of elliptic homogenization problems in nondivergence-form

Timo Sprekeler[1] Yves Capdeboscq[2] Endre Süli[3]

We use uniform $W^{2,p}$ estimates to obtain corrector results for periodic homogenization problems of the form $A(x/\varepsilon) : D^2u_\varepsilon = f$ subject to a homogeneous Dirichlet boundary condition. We propose and rigorously analyze a numerical scheme based on finite element approximations for such nondivergence-form homogenization problems. The second part of this work focuses on the approximation of the corrector and numerical homogenization for the case of nonuniformly oscillating coefficients. Numerical experiments demonstrate the performance of the scheme.

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Notes on session “Numerical Analysis”

Stress approximations in solid mechanics

Fleurianne Bertrand[1]

The usual way to perform computations in solid mechanics is based on the representation of a primal variable, commonly the displacement or the pressure, by suitable finite element spaces. From such a finite element approximation, other variables like stresses can be reconstructed in a post-processing step. This primal formulation had great success of practical computational engineering for advanced problems in solid mechanics. However, it is well-known that the loss of accuracy due to the reconstruction step can lead to non-physical solutions, in particular for the simulation of challenging problems like in fracture mechanics or coupled problems, or for complex boundary value problems with discontinuities, strong anisotropy, heterogeneity, or incompressibility.

In order to master these challenges and develop robust numerical schemes, an accurate representation of the stress-field is needed. To this end, the associated first-order system can be considered, such that the stress-field and the primal variable are both part of the model. This system allow the use of variational formulations involving the stresses as independent unknowns, which are approximated simultaneously to the variable of the primal approach. The standard mixed finite element approach leads then to a saddle-point problem, in which the momentum balance is approximated in an optimal way, if appropriate finite element combinations are used. However, the construction of those finite element spaces is challenging, since a stability condition between the mixed spaces has to be established.

In order to avoid the complexity of this stability condition, a positive definite system can be obtained by minimizing the residuals in the partial differential equations. Using the L^2-norm leads to the Least-Squares method, which provides the advantage of an inherent a posteriori error estimator. However, there is a strong connection of this stress approximation to that obtained from a mixed formulation. In fact, the error associated with the momentum balance can be proved to be of higher order than the overall error of the least-squares approach. This implies that the favorable conservation properties of the dual-based mixed methods and the error control of the least squares method can be combined.

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Optimal convergence rates in L^2 for a first order system least-squares Finite Element Method

Maximilian Bernkopf[1] Jens Markus Melenk[2]

We consider a Poisson-like second order model problem written as a system of first order equations. For the discretization an $\boldsymbol{H}(\Omega, \mathrm{div}) \times H^1(\Omega)$- conforming least squares formulation is employed. A least squares formulation has the major advantage that regardless of the original formulation the linear system resulting from a least squares type discretization is always positive semi-definite, making it easier to solve. Even though our model problem in its standard $H^1(\Omega)$ formulation is coercive our methods and lines of proof can most certainly be applied to other problems as well, see [2, 3] for an application to the Helmholtz equation. A major drawback of a least squares formulation is that the energy norm is somewhat intractable. Deducing error estimates in other norms, e.g., the $L^2(\Omega)$ norm of the scalar variable, is more difficult. Numerical examples in our previous work [2] suggested convergence rates previous results did not cover. Closing this gap in the literature will be the main focus of the talk. To that end we showcase a duality argument in order to derive L^2 error estimates of the scalar variable, which was the best available estimate in the literature. We then perform a more detailed analysis of the corresponding error terms. This analysis then leads to optimal convergence rates of the method. The above procedure can then be applied to more complicated boundary conditions, for which an analogous result is a nontrivial task. As a tool, which is of independent interest, we develop $\boldsymbol{H}(\Omega, \mathrm{div})$-conforming approximation operators satisfying certain orthogonality relations. For the analysis, a crucial tool are recently developed projection based commuting diagram operators, see [4].

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Notes on session “Least-Squares Finite Element Method”

Least-squares formulations with application to FE-simulations of fluid-structure interaction problems

Solveigh Averweg[1] Alexander Schwarz[2] Carina Nisters[3] Jörg Schröder[4]

This contribution deals with an approach to model fluid-structure interaction problems with monolithic coupling. The discretization of the fluid and the solid domain is based on the least-squares finite element method (LSFEM), whose application results in a minimization problem with symmetric positive definite equation systems also for non self-adjoint problems. The resulting second-order systems are reduced to first-order systems by introducing new variables, leading to least-squares formulations for both domains based on the stresses and velocities as presented in e.g. [1] and [2]. A conforming discretization of the unknown fields in H^1 and H(div) using Lagrange interpolation polynomials and vector-valued Raviart-Thomas interpolations functions involves to the automatic fulfillment of the coupling conditions. In more detail, a discretization in H^1 ensures continuity of the velocity field and a discretization in H(div) results in continuity of the normal stress components at the interface. The governing equations are based on the incompressible Navier-Stokes equations in an Arbitrary-Lagrangian-Eulerian (ALE) framework for the fluid domain and on linear elastodynamics for the solid domain.

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Least-squares Finite Element Methods for sea ice modelling

Steffen Münzenmaier[1] Fleurianne Bertrand[2]

In this talk a viscoplastic sea ice model is considered. Sea ice is modelled as a generalized Newtonian compressible fluid, which satisfies a power law. The nonlinearity caused by the power law for the viscosity is severe and requires careful treatment. The talk considers a first-order system least squares method (FOSLS), where the velocity, ice height, ice concentration and the stress are approximated. This approach leads to a nonlinear system of partial differential equations and for the least-squares finite element method to a discrete non-quadratic minimization problem.

The talk addresses the numerical challenges that arise while solving this system. After deriving a proper linearization and discussing fitting approximation spaces, numerical experiments will be presented. Furthermore theoretical results considering the well posedness of the problem will be discussed. The talk also examines the advantages of adaptive refinement in this setting.

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Symbolic evaluation of hp-FEM element matrices on simplices

Tim Haubold[1] Veronika Pillwein[2] Sven Beuchler[3]

In this talk we consider high-order finite element discretizations of linear elliptic boundary value problems. Following e.g. [Beuchler et al., 2012 [1], Karniadakis, Sherwin [2]] a set of hierarchic basis functions on simplices is chosen. For an affine simplicial triangulation this leads to a sparse stiffness matrix. Moreover the L_2-inner product of the interior basis functions is sparse with respect to the polynomial order p. The construction relies on a tensor-product based construction with properly weighted Jacobi polynomials.
In this talk we present algorithms which compute the remaining non zero entries of mass- and stiffness matrix in optimal arithmetical complexity. In order to obtain this result, recursion formulas based on symbolic methods [3] are used. The presented techniques can be applied not only to scalar elliptic problems in H^1 but also for vector valued problems in $H(\mathrm{div})$ and $H(\mathrm{curl})$, where an explicit splitting of the higher-order basis functions into solenoidal and non-solenoidal ones is used.

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A locally modified high order Finite Element Method for interface problems

Gozel Judakova[1] Thomas Richter[2] Stefan Frei[3]

We consider the locally modified finite element method [1] for the Laplace equation with a discontinuous coefficient across an interface line, which is able to resolve weak discontinuities in interface problems. Our approach is based on a mixed triangular-quadrilateral mesh as in a fitted finite element method. If a patch of elements is cut by the interface, we start by dividing the patch into four quadrilaterals whose boundaries approximate the interface. Each quadrilateral is then split into two triangles. We show that all interior angles of the triangles are bounded by $117°$ independent of the position of the interface on an edge. We present some numerical examples to show optimal order of convergence for elliptic problems.

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Notes on session "Least-Squares Finite Element Method and High Order"

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