Higher order Galerkin-collocation time discretization for the Navier-Stokes equations with Nitsche's method

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We present families of families of Galerkin–collocation time discretization schemes for the incompressible Navier–Stokes equations

$$\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} - \nu \Delta \boldsymbol{v} + \nabla p = \boldsymbol{f}\,, \quad \nabla \cdot \boldsymbol{v} = 0\,.$$

The conceptual basis of these schemes is the establishment of a direct connection between the Galerkin method and the classical collocation methods, with the perspective of achieving the accuracy of the former with reduced computational costs in terms of less complex algebraic systems of the latter. Higher regularity in time of the discrete solutions is also ensured; cf. [1,3].

As a further ingredient of our approach, we employ the Nitsche method to impose all types of boundary conditions in a weak form. Thereby, the essential building block for capturing problems of fluid-structure interaction in the future is provided by our approach. For an application of our discretization techniques to elastodynamics and wave problems we refer to [1,2].

References:

[1] M. Anselmann, M. Bause, S. Becher, G. Matthies, *Galerkin–collocation approximation in time for wave equations*, **to appear** (2019), pp. 1–25

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