Stress approximations in solid mechanics

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The usual way to perform computations in solid mechanics is based on the representation of a primal variable, commonly the displacement or the pressure, by suitable finite element spaces. From such a finite element approximation, other variables like stresses can be reconstructed in a post-processing step. This primal formulation had great success of practical computational engineering for advanced problems in solid mechanics. However, it is well-known that the loss of accuracy due to the reconstruction step can lead to non-physical solutions, in particular for the simulation of challenging problems like in fracture mechanics or coupled problems, or for complex boundary value problems with discontinuities, strong anisotropy, heterogeneity, or incompressibility.

In order to master these challenges and develop robust numerical schemes, an accurate representation of the stress-field is needed. To this end, the associated first-order system can be considered, such that the stress-field and the primal variable are both part of the model. This system allow the use of variational formulations involving the stresses as independent unknowns, which are approximated simultaneously to the variable of the primal approach. The standard mixed finite element approach leads then to a saddle-point problem, in which the momentum balance is approximated in an optimal way, if appropriate finite element combinations are used. However, the construction of those finite element spaces is challenging, since a stability condition between the mixed spaces has to be established.

In order to avoid the complexity of this stability condition, a positive definite system can be obtained by minimizing the residuals in the partial differential equations. Using the L^2-norm leads to the Least-Squares method, which provides the advantage of an inherent a posteriori error estimator. However, there is a strong connection of this stress approximation to that obtained from a mixed formulation. In fact, the error associated with the momentum balance can be proved to be of higher order than the overall error of the least-squares approach. This implies that the favorable conservation properties of the dual-based mixed methods and the error control of the least squares method can be combined.

References:

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