An accuracy condition for the finite element discretization of Biot's equations

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Biot's equations are a system of time-dependent partial differential equations, which describe the mechanical behaviour of fluid-saturated porous media [1]. The unknowns of such a system are the solid displacement and in the pore pressure.

Finite element discretizations of Biot's equations may exhibit unphysical oscillations in the solid displacement and in the pore pressure, which tend to increase when the time step is reduced [2]. In the one-dimensional case, a lower limit for the time step has been derived, in terms of the mesh size and the mechanical properties.

Differently from previous works, e.g. [2, 3], we identify the unphysical oscillations as a violation of a generalized discrete maximum principle and, in this way, we extend the lower limit for the time-step to the two- and three-dimensional cases [4].

References:

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