A modified Arnoldi algorithm to compute transmission eigenvalue of an interior transmission eigenvalue problem

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The inverse scattering plays a crucial role in the modern field of partial differential equations. It is a special field of interest for many mathematicians who deal with partial differential equations theory and it is in continuous process. The nature of the inverse problem is generally ill-posed, more specifically the third condition fails (the solution does not depend on the initial data), so it is a real challenge to find a solution to the given problem. In this paper we deal with an interior transmission problem which is a boundary value problem compounded of two partial differential equations of second order defined in a bounded domain that correspond to the scatterer. Its homogeneous version is referred to as the transmission eigenvalue problem, which is nonlinear and non self-adjoint eigenvalue problem. After proving the discreetness and the existence of the following problem

$$\begin{aligned} &\Delta w + k^2 \;\; nw = 0 \;\text{ in } \Omega \\ &\Delta v + k^2 \;\; v = 0 \;\text{ in } \Omega \\ &w - v = -\eta \;\; \partial v/\partial \vartheta \;\text{ in } \partial\Omega \\ &\partial w/\partial \vartheta - \partial v/\partial \vartheta = 0 \;\text{ in } \partial\Omega \end{aligned}$$

we focus in a numerical method that gives the first value of k. (the first eigenvalue) . More precisely, we use the finite element method. We transform the transmission problem into a weak problem. Then we use standard piecewise linear finite elements to discretize this problem. We need only a few lowest real values of transmission eigenvalue in inverse scattering theory, so a modified algorithm using Arnoldi method is used to compute these eigenvalues. We use Matlab for the implementation because it is more convenient to use the finite element method.

References:

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