Quasi-best approximation in PDE constraint optimization

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We consider finite element solutions to quadratic optimization problems, where the state depends on the control via a well-posed linear partial differential equation. Exploiting the structure of a suitably reduced optimality system, we prove that the combined error in the state and adjoint state of the variational discretization on FEM spaces is bounded by the best approximation error in the underlying discrete spaces. The constant in this bound depends on the inverse square-root of the Tikhonov regularization parameter. Furthermore, if the operators of control-action and observation are compact, this quasi-best-approximation constant becomes independent of the Tikhonov parameter as the meshsize tends to 0 and we give quantitative relationships between meshsize and Tikhonov parameter ensuring this independence. We also derive generalizations of these results when the control variable is discretized or when it is taken from a convex set.

References:

[1] https://arxiv.org/abs/1904.07049

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