Symbolic evaluation of hp-FEM element matrices on simplices

Tim Haubold[1] Veronika Pillwein[2] Sven Beuchler[3]

In this talk we consider high-order finite element discretizations of linear elliptic boundary value problems. Following e.g. [Beuchler et al., 2012 [1], Karniadakis, Sherwin [2]] a set of hierarchic basis functions on simplices is chosen. For an affine simplicial triangulation this leads to a sparse stiffness matrix. Moreover the L_2-inner product of the interior basis functions is sparse with respect to the polynomial order p. The construction relies on a tensor-product based construction with properly weighted Jacobi polynomials.
In this talk we present algorithms which compute the remaining non zero entries of mass- and stiffness matrix in optimal arithmetical complexity. In order to obtain this result, recursion formulas based on symbolic methods [3] are used. The presented techniques can be applied not only to scalar elliptic problems in H^1 but also for vector valued problems in $H(\mathrm{div})$ and $H(\mathrm{curl})$, where an explicit splitting of the higher-order basis functions into solenoidal and non-solenoidal ones is used.

References:

[1] Beuchler, Pillwein, Schöberl, Zaglmayr, *Sparsity Optimized High Order Finite Element Functions on Simplices* (2012)

[2] Karniadakis, Sherwin, *Spectral/HP Element Methods for CFD*, (1999)

[3] Manuel Kauers, Guess: A Mathematica package for guessing multivariate recurrence equations

[1]IfAM, Leibniz University Hannover
Haubold@ifam.uni-hannover.de

[2]RISC, Johannes Kepler University Linz
veronika.pillwein@risc.jku.at

[3]IfAM, Leibniz University Hannover
beuchler@ifam.uni-hannover.de