A general framework for applying the DWR method on variational inequalities

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In this talk, we consider variational inequalities of first and second kind including a smooth nonlinear differential operator. We rewrite the variational inequality with the help of nonlinear complementarity (NCP) functions as a nonlinear problem. However, due to the nonsmoothness of the NCP functions, the resulting problem is not smooth. Furthermore we have potentially to assume additional properties of the analytic solution to do the reformulation. We want to apply the dual weighted residual (DWR) method to estimate the discretization error in a user defined quantity of interest. Due to the nonsmoothness of our formulation, we cannot directly apply the classic DWR approach, where the problem is linearized on the basis of the directional derivative of the underlying semi linearform. Instead, we use the active sets provided by the NCP functions to do the linearization w.r.t. them, while the differential operator is treated in the classic way. The arising dual problem is closely connected to the linear system of equations, which has to be solved in the last step of a semi smooth Newton method applied to the original problem. We derive an error identity, which consists in the primal residual, the dual residual and a remainder term. The remainder term is neglectable. The primal residual and the dual residual are weighted by the dual and the primal discretization error, respectively. Thus, their evaluation has to be approximated numerically. Finally, the error estimate is localized to the mesh cells by a filtering approach to utilize it in an adaptive strategy. As an example for the application of this framework we discuss Signorini's problem with friction discretized by a mixed method. Finally, we give an outlook on thermoplastic contact problems.

References:

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