Space-time Finite Element Methods for parabolic initial-boundary value problems with non-smooth solutions

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We consider locally stabilized, conforming finite element schemes on completely unstructured simplicial space-time meshes for the numerical solution of parabolic initial-boundary value problems with variable, possibly discontinuous in space and time coefficients. Discontinuous coefficients, non-smooth boundaries, changing boundary conditions, non-smooth or incompatible initial conditions, and non-smooth right-hand sides can lead to non-smooth solutions. For instance, in electromagnetics, permanent magnets cause line-delta-distributions in the source term in 2d quasi-magnetostatic simulations of electrical machines.

We present new a priori and a posteriori error estimates for low-regularity solutions. In order to avoid reduced convergence rates appearing in the case of uniform mesh refinement, we also consider adaptive refinement procedures based on residual a posteriori error indicators and functional a posteriori error estimators. The latter provides guaranteed upper bounds on the error. The huge system of space-time finite element equations is then solved by means of GMRES preconditioned by space-time algebraic multigrid. In particular, in the 4d space-time case that is 3d in space, simultaneous space-time parallelization can considerably reduce the computational time. We present and discuss numerical results for several examples possessing different regularity features. The implementation is performed within MFEM.

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