Solving generalized KdV-Burgers' equations using a hybridized discontinuous Galerkin method

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In this talk, we aim to proceed to construct and analyze a hybridized discontinuous Galerkin (HDG) method for solving generalized KdV-Burgers' (KdVB) equations. Following KdVB equation is considered

$$\mathfrak{u}_t + f(\mathfrak{u})_x - (a(\mathfrak{u})\mathfrak{u}_x)_x + (r'(\mathfrak{u})g(r(\mathfrak{u})_x)_x)_x = 0, \qquad x \in \Omega \subset \mathbb{R}, \qquad t \in (0,T] \tag{1}$$

where f, a, r and g are some given functions. For the sake of simplicity, we consider a special case of equation (1) as follows

$$\mathfrak{u}_t + f(\mathfrak{u})_x - \alpha_1 \mathfrak{u}_{xx} + \alpha_2 \mathfrak{u}_{xxx} = 0, \qquad x \in \Omega \subset \mathbb{R}, \qquad t \in (0,T] \tag{2}$$

in which $\alpha_1 > 0$, $\alpha_2 \in \mathbb{R}$, and f is a suitable function which is in most problems as $f(\mathfrak{u}) = \frac{\alpha_0}{n+1}\mathfrak{u}^{n+1}$. We focus on equation (2) because it is more applicable in engineering and physics in particular it has many applications in plasmas and fluids. On the other hand, the HDG method is one of the outstanding and successful methods for solving nonlinear evolution equations. Hence, we exploit an HDG discretization in space and an Euler method in time. A theorem related to the stability of the proposed method is proved. More precisely, we prove that if equation (2) is equipped with periodic or appropriate homogeneous Dirichlet boundary conditions then the proposed HDG method is stable subject to proper choice of stabilization parameters. We examine some different examples to observe optimal convergence in both approximate solution and its derivative. Propagation and interaction of some solitons such as the bell-type and/or kink-type and the evolution of the shock-wave solutions are tested to demonstrate the effectiveness and applicability of the proposed method.

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