Formation of wrinkles on a coated elastic substrate

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We consider a thin elastic substrate coated with a hard film that forms wrinkles when stretched or sheared. The elastic substrate is modeled using a hyperelastic, homogeneous and isotropic material. The film is modeled using a Cosserat shell as proposed by Neff. This involves a microrotation field of orthonormal director triads on the coated part of the substrate boundary, which will be denoted by Ω_ξ.
The material behavior can be described by the sum of two energies:

$$J(\varphi, R) = \int_\Omega W_e(\nabla\varphi)\, dV + \int_{\Omega_\xi} W_c(\varphi_{|\Omega_\xi}, R)\, dS$$

where $W_e : GL^+(3) \to \mathbb{R}$ is the elastic energy density and $W_c : GL^+(2) \times SO(3) \to \mathbb{R}$ is the Cosserat shell energy density depending on the surface deformation function φ and the microrotation field R.
We discretize the problem using finite elements for the substrate displacement and geodesic finite elements for the microrotation field. Geodesic finite elements are a generalization of standard finite elements to spaces of functions mapping into a Riemannian manifold.
We obtain an algebraic optimization problem on a finite-dimensional Riemannian manifold. To solve this problem we propose a distributed Riemannian trust-region solver with a monotone multigrid method for the constraint inner problems. Numerical experiments show that we can efficiently reproduce wrinkling patterns of the coupled system.

References:

[1] https://arxiv.org/pdf/1412.3668.pdf

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