Stable space-time Finite Elements for viscous flows

Douglas Ramalho Queiroz Pacheco[1] Olaf Steinbach[2]

Space-time finite element formulations differ from conventional methods by treating time and space in a similar manner. This means that time is considered as an additional dimension, so that the whole space-time domain is discretized with finite elements. Such formulations have proven attractive for the solution of various problems, for instance, by offering a natural framework to tackle flow problems with time-dependent domains. In the present work, the well-known concept of the Taylor-Hood element is extended for the Galerkin space-time variational formulation of the Stokes system. Classical spatial Taylor-Hood elements are constructed by interpolating flow velocities with a one order higher polynomial degree than the pressure. This guarantees the fulfillment of a discrete inf-sup condition, thereby yielding a stable method. As our main contribution, we construct space-time Taylor-Hood elements by using second-order interpolation for the velocity and first-order interpolation for the pressure – with the same degree for space and time. This yields an unconditionally stable and optimally convergent method. We further consider two types of elements, namely, prismatic (simplicial in space, tensor product in time) and tetrahedral (fully simplicial). A comparative study between these two different Taylor-Hood-type elements is carried out, and numerical simulations reveal the potential of the method.

[1]Graz University of Technology, Institute of Applied Mathematics, Graz, Austria
pacheco@math.tugraz.at

[2]Graz University of Technology, Institute of Applied Mathematics, Graz, Austria
o.steinbach@tugraz.at