The maximal angle condition on finite elements – useful or not?

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There are several assumptions on the shape of triangular or tetrahedral finite elements around. The majority of papers is written for shape regular elements (minimal angle condition). Less restrictive is the regular vertex property: there must be one vertex where the directions of the adjacent edges form a stable coordinate system. In two dimensions, this property is equivalent to the maximal angle condition. In three dimensions, however the maximal angle condition is weaker than the regular vertex property.

Interpolation error estimates for elements without shape regularity may be written in different quality, possibly using one or another of the above shape conditions. We give an overview of such results. In the energy norm, the order of the interpolation error is usually transferred to the order of the discretization error of the finite element method. But there are examples where the finite element error is of higher order than the interpolation error. The finite element method may converge in situations where the interpolation error does not; in particular meshes violating the maximal angle condition were discussed in the literature recently. However, there is also an example where the finite element method does not converge to the exact solution of the problem. With a numerical study we stress that the finite element method converges; it converges to a function which is not the exact solution. Hence a numerical test with the error measure as the difference to a solution on a very fine mesh of the same family leads to completely wrong conclusions: one observes a convergence order.

The necessity of the maximal angle condition could also be discussed in a posteriori analysis. We report also here on numerical test results.