Efficient solution of parameter identification problems with H^1-regularization

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We consider parameter identification problems for parameters distributed in space under H^1 regularization. Linearization using the Gauss-Newton method leads to problems to be solved in each step that can be cast as boundary value problems involving a low-rank modification of the Laplacian. Using algebraic multigrid methods as a fast Laplace solver, the Sherman-Morrison-Woodbury formula can be employed to construct an efficient preconditioner for these linear problems.

First, we will develop this method in the functional framework, thus obtaining a clear methodology for selection of boundary conditions that arise from the H^1 regularization. Second, we construct a method for solution of the discretized linear systems, which is based on any existing fast Poisson solver and the Woodbury formula. Last but not least, we demonstrate efficacy of the method with scaling experiments. We consider electrical resistivity tomography, i.e., identification of spatial distribution of electrical resistivity in the medium from a finite number of point-to-point voltage measurements under point-to-point DC current excitations. This is a non-linear inverse problem. We will show how the performance of the proposed method scales with complexity of the problem. Note it is not only the mesh parameter, which drives the scaling, but also the number of observations, which is related to the rank of the low-rank modification. We construct a series of resistivity distributions which require refinement in one of the two or both parameters and we show how the proposed method scales.