From Design to Numerical Analysis of Partial Differential Equations: A Unified Mathematical Framework

Annalisa Buffa

Computer-based simulation of partial differential equations (PDEs) involves approximating the unknowns and relies on suitable description of geometrical entities such as the computational domain and its properties. The Finite Element Method (FEM) is by large the most popular technique for the computer-based simulation of PDEs and hinges on the assumption that discretized domain and unknown fields are both represented by piecewise polynomials, on tetrahedral or hexahedral partitions. In reality, the simulation of PDEs is a brick within a workflow where, at the beginning, the geometrical entities are created, described and manipulated with a geometry processor, often through Computer-Aided Design systems (CAD), and then used for the simulation of the mechanical behaviour of the designed object. This workflow is often repeated many times as part of a shape optimisation loop. Within this loop, the use of FEM on CAD geometries (which are mainly represented through their boundaries) calls then for (re-)meshing and re-interpolation techniques that often require human intervention resulting in inaccurate solutions and lack of robustness of the whole process. In my talk, I will present the mathematical counterpart of this problem, I will discuss the mismatch in the mathematical representations of geometries and PDEs unknowns and introduce a promising framework where geometric objects and PDEs unknowns are represented in a compatible way. Within this framework, the challenges to be addressed in order to construct robust PDE solvers are many and I will discuss some of them. Mathematical results will be supported by numerical validation.