Pressure-robust discretization of the Stokes equations on domains with edges

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Due to the relaxation of the divergence constraint on the discrete level, most classical finite element methods for the Stokes equations do not replicate the following fundamental property of the continuous problem: the velocity solution is not affected by perturbations of the right hand side data in form of gradient fields. Discretizations that manage to transfer this property to the discrete setting are called pressure-robust. A reconstruction approach using an interpolation operator on the test functions in the linear form, mapping them to exactly divergence-free functions in the sense of $\mathbf{H}(\mathrm{div})$, can recover the pressure-robustness property for almost all classical mixed methods for incompressible flows.

In addition to the above difficulty when discretizing the Stokes equations, the solution also has locally singular behavior near concave edges, resulting in generally degraded convergence rates on quasi-uniform meshes. Anisotropic mesh grading towards these edges has proved to recover the optimal convergence orders. Using the reconstruction technique, we show a pressure-robust, optimal order error estimate for a discretization that is uniformly inf-sup stable. Some numerical examples support these results.