A posteriori error estimation on anisotropic meshes

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Solutions of partial differential equations frequently exhibit corner singularities and/or sharp boundary and interior layers. To obtain reliable numerical approximations of such solutions in an efficient way, one may want to use meshes that are adapted to solution singularities. Such meshes can be constructed using a priori information on the exact solutions, however it is rarely available in real-life applications. Therefore the automated mesh construction by adaptive techniques is frequently employed. This approach requires no initial asymptotic understanding of the nature of the solutions and the solution singularity locations. Note that reliable adaptive algorithms are based on a posteriori error estimates, i.e. estimates of the error in terms of values obtained in the computation process: computed solution and current mesh. Such a posteriori error estimates for elliptic partial differential equations will be the subject of this talk.

Our main goal in this talk is to present residual-type a posteriori error estimates in the maximum norm, as well as in the energy norm, on anisotropic meshes, i.e. we allow mesh elements to have extremely high aspect ratios. The error constants in these estimates are independent of the diameters and the aspect ratios of mesh elements. Note also that, in contrast to some a posteriori error estimates on anisotropic meshes in the literature, our error constants do not involve so-called matching functions (that depend on the unknown error and, in general, may be as large as mesh aspect ratios).

To deal with anisotropic elements, a number of technical issues have been addressed. For example, an inspection of standard proofs for shape-regular meshes reveals that one obstacle in extending them to anisotropic meshes lies in the application of a scaled traced theorem when estimating the jump residual terms (this causes the mesh aspect ratios to appear in the estimator). For the estimation in the energy norm, a special quasi-interpolation operator is constructed on anisotropic meshes, which may be of independent interest.

In the second part of the talk, the focus will shift to the efficiency of energy-norm error estimators on anisotropic meshes. A simple numerical example will be given that clearly demonstrates that the standard bubble function approach does not yield sharp lower error bounds. To be more precise, short-edge jump residual terms in such bounds are not sharp. To remedy this, we shall present a new upper bound for the short-edge jump residual terms of the estimator.

We shall also touch on that certain perceptions need to be adjusted for the case of anisotropic meshes. In particular, it is not always the case that the computed-solution error in the maximum norm is closely related to the corresponding interpolation error.

References

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