Stable and robust finite element discretisations for incompressible flow problems

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The development and analysis of stable and robust finite element methods for incompressible flow problems remains to be a great challenges in numerical analysis. Several severe problems have to be handled: incompressibility constraint, nonlinear and dominating convection, pressure.

Our talk is two-fold. First, we address local projection stabilisation methods that provide stable velocity approximations. Second, we discuss schemes that ensure robust pressure approximations.

The local projection methods have been first designed for equal-order interpolation of velocity and pressure to allow a separate stabilization of the velocity, pressure, and incompressibility constraints. Introduced for the Stokes equations, they were extended to transport problems and Oseen problems with equal-order interpolation. Local projection stabilisations applied to inf–sup stable elements without skipping the pressure stabilization followed. Originally, the local projection technique was proposed as a two-level method where the quantities of interest are locally projected onto discontinuous finite element spaces living on a coarser mesh. However, a general approach allows to construct local projection methods where approximation and projection spaces are defined on the same mesh. We present an overview on local projection schemes for both equal-order interpolation and inf-sup stable finite element pairs.

In incompressible flows with vanishing normal velocities at the boundary, irrotational forces in the momentum equations are balanced completely by the pressure gradient. Unfortunately, nearly all mixed discretisations in the primal variables velocity and pressure violate this property. to overcome this drawback, we modify the discrete problem of well-established and well-understood classical mixed methods by introducing a velocity reconstruction operator that maps discretely divergence-free functions to divergence-free ones. We consider the case of non-affine meshes and discuss the impact of the Piola transform for defining appropriate discrete function spaces.