Analysis of a finite element method based on L-type formulae for solving subdiffusion equations

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We consider the two-dimensional time-fractional (subdiffusion) equation with the Caputo fractional derivative of order $\alpha \in (0, 1)$, which is approximated usually by using some formulae known as L1, L1-2, and L1-2-3. We call them L-type formulae. These formulae are obtained by applying the Lagrange interpolation method. Based on the idea of B-spline interpolation, a new class of recipes called S1, S2, and S3 have been constructed successfully in [3]. We call them S-type formulae. In our previous work [2], based on L-type and new S-type formulae for approximating the time-fractional derivative and finite element method (FEM) or finite difference method (FDM) for the spatial discretization, some high-order fully discrete schemes were proposed to solve the problem. Furthermore, the convergence order of the proposed schemes in one and two-dimensional spaces investigated numerically.

What is still lacking is some theoretical results. Analysis of the FDM based on L-type formulae has been carried out recently in [1]. Similarly, we aim to prove that the fully discrete schemes based on FEM and L-type methods are unconditionally stable concerning the L^2-norm. It will also be analyzed the convergence of the proposed fully-discrete schemes.

References

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[3] M. Ramezani, R. Mokhtari, G. Haase, Some high order formulae for approximating Caputo fractional derivatives, Applied Numerical Mathematics, 153, 300-318, 2020.