

# Goal-oriented Adaptivity for the Diffusion Equation with Point Sources and Point Observations

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We consider the source-to-observation map relating the effect of a point source on point evaluations of the stationary diffusion equation as arises in *geoelectrical resistivity tomography*, an inverse problem for reconstructing the distribution of electrical conductivity in a medium based on a series of point observations under point loads.

We reformulate the quantity of interest using the *adjoint method* and the *subtraction method*. The role of the Dirac distributions representing the point load and the point observations is fully symmetric, reflecting the duality. The subtraction method is used to characterize the behavior of the primal and adjoint solutions in the vicinity of the point singularity. This formulation enables the use of *the method of equilibrated fluxes* to construct an a posteriori error estimator which is fully reliable and locally computable. The estimator is used to drive adaptive mesh refinement. We demonstrate the efficacy of the method through numerical experiments.

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