Stochastic Discontinuous Galerkin Methods with Low-Rank Solvers for Convection Diffusion Equations

Pelin Çiloğlu[1] Hamdullah Yücel[2]

To simulate complex kinds of behavior in physical systems, one makes predictions and hypotheses about certain outputs of interest with the help of simulation of mathematical models. However, due to the lack of knowledge or inherent variability in the model parameters, such real-problems formulated by mathematical models generally come with uncertainty concerning computed quantities. Therefore, the idea of uncertainty quantification (UQ) has become a powerful tool for modeling physical phenomena in the last few years.

In this talk, we investigate numerical behaviour of a convection diffusion equation with random coefficients by approximating statistical moments of the solution. Stochastic Galerkin approach, turning the original stochastic problem to a system of deterministic convection diffusion equations, is used to handle the stochastic domain, whereas discontinuous Galerkin method is used to discretize spatial domain due to its local mass conservativity. To address the curse of dimensionality of Stochastic Galerkin method, we take advantage of the low−rank Krylov subspace methods, which reduce both the storage requirements and the computational complexity by exploiting a Kronecker−product structure of system matrices. The efficiency of the proposed methodology is illustrated by numerical experiments on the benchmark problems.

References:

[1] P. Çiloğlu, H. Yücel, Stochastic discontinuous Galerkin methods with low-rank solvers for convection-diffusion equations, ArXiv e-prints arXiv:2011.02443, 2020

[1]Middle East Technical University, Institute of Applied Mathematics
pciloglu@metu.edu.tr

[2]Middle East Technical University, Institute of Applied Mathematics
yucelh@metu.edu.tr