

# Bayesian inversion for unified ductile phase-field fracture

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The prediction of crack initiation and propagation in ductile failure processes are challenging tasks for the design and fabrication of metallic materials and structures on a large scale. Numerical aspects of ductile failure dictate a sub-optimal calibration of plasticity- and fracture-related parameters for a large number of material properties. These parameters enter the system of PDEs as a forward model. Thus, an accurate estimation of the material parameters enables the precise determination of the material response in different stages, particularly for the post-yielding regime, where crack initiation and propagation take place. We develop a Bayesian inversion framework for ductile fracture to provide accurate knowledge regarding effective mechanical parameters. For this, synthetic and experimental observations are used to estimate the posterior density of the unknowns. To model the ductile failure behavior of solid materials, we rely on the phase-field approach to fracture, for which we present a unified formulation that allows recovering different models on a variational basis. In the variational framework, incremental minimization principles for a class of gradient-type dissipative materials are used to derive the governing equations. The overall formulation is revisited and extended to the case of anisotropic ductile fracture. Three different models are subsequently recovered by certain choices of parameters and constitutive functions, which are later assessed through Bayesian inversion techniques. A step-wise Bayesian inversion method is proposed to determine the posterior density of the material unknowns for a ductile phase-field fracture process. To estimate the posterior density function of ductile material parameters, three common MCMC techniques are employed: (i) the Metropolis-Hastings algorithm, (ii) delayed-rejection adaptive Metropolis, and (iii) ensemble Kalman filter combined with MCMC. To examine the computational efficiency of the MCMC methods, we employ the  $\hat{R} - convergence$  tool. The resulting framework is algorithmically described in detail and substantiated with numerical examples.

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