A least-squares Galerkin approach to gradient recovery for Hamilton-Jacobi-Bellman equation with Oblique derivative boundary conditions

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In this presentation, we discuss a numerical solution of the oblique derivative boundary-value Hamilton-Jacobi-Bellman(HJB) problem of the form

$$\sup_{\alpha\in\mathcal{A}} \left(\mathbf{A}^\alpha : D^2u - f^\alpha\right) = 0 \quad \text{in } \Omega, \quad \text{and} \quad \nabla u \cdot \ell \text{ is constant on } \partial\Omega,$$

where $\mathcal{A}$ is a compact metric space, $\Omega \subset \mathbb{R}^2$ is a curved domain, $\mathbf{A} \in L^\infty(\Omega; C^0(\mathcal{A}; \mathrm{sym}(\mathbb{R}^{d\times d})))$, D^2u denote the Hessian of u, $f \in L^2(\Omega; C^0(\mathcal{A}))$ and ℓ is a given unit oblique vector field. Since the HJB operator is not differentiable in the usual sense, the classical Newton linearization can not be used here. Mousavi et al. have shown that the HJB operator that is considered from the infinite-dimensional space $H^2(\Omega)$ to $L^2(\Omega)$, is Newton differentiable. Thus, the door is open to apply the semi-smooth Newton linearization approach to linearize fully nonlinear HJB equations at the continuous level (before discretization). Via this linearization, the HJB equation turns to a recursive linear problem in nondivergence form. To tackle linear equations in non-divergence form in each iteration, the least-squares approach involving gradient recovery, presented by Lakkis and Mousavi, is used. Indeed, corresponding to the linear equation in non-divergence form, we introduce a quadratic cost functional which provides a setting to impose constraints of the unknowns, boundary values and compatibility condition through adding some square terms to that. Then, we consider the Euler-Lagrange equation of the minimization problem and approximate its solution by the standard finite element method in subspaces of $H^1(\Omega)$. We present a priori and a posteriori error analysis of the method and exhibit numerical experiments which support the theoretical findings.

References:

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