Algebraic error and residual-based a posteriori error estimator

Jan Papež[1] Zdeněk Strakoš[2]

In the talk, we will discuss a widely used residual-based error estimator that has been derived for the Galerkin solution, i.e. asuming the exact solution of the associated algebraic system. We will see that a generalization of the estimator in order to be used for a computed, inexact approximation requires a careful analysis and it results in a form with an additional unknown multiplicative factor. We will also illustrate how an adaptive mesh refiment based on this error estimator can be affected when the estimator is evaluated for the Galerkin solution or a computed approximation.

We use these results to demonstrate more general messages: (i) Results based on the assumption of exact algebraic solution should not be used for computed approximations. A derivation (or revision) of theoretical results that take into account inexact algebraic solution can be more difficult and/or the results might be weaker. (ii) An efficient solution procedure requires thorough understanding and interaction between all phases of the solution, such as discretization, preconditioning, algebraic solution, and error estimation.

References:

[1] J. Papež and Z. Strakoš: On a residual-based a posteriori error estimator for the total error. IMA Journal of Numerical Analysis, 38(3):1164–1184, 2017.

[2] J. Papež: Algebraic Error in Matrix Computations in the Context of Numerical Solution of Partial Differential Equations. PhD thesis, Charles University, Prague, November 2016.

[1]Czech Academy of Sciences, Institute of Mathematics
papez@math.cas.cz

[2]Charles University, Prague, Czech Republic