$\mathcal{H}$-Matrix approximations to inverses for FEM-BEM couplings

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$\mathcal{H}$-matrices are blockwise low-rank matrices of rank r where the blocks are organized in a tree $\mathbb{T}$ (generated by the standard admissibility condition) so that the memory requirement is typically $O(Nr\text{depth}(\mathbb{T}))$, where N is the problem size. This format comes with an (approximate) arithmetic that allows for addition, multiplication, inversion, and LU-factorization in logarithmic-linear complexity. Therefore, computing an (approximate) inverse in the $\mathcal{H}$-format can be considered an effective alternative to a direct solver or it can be used as a "black box" preconditioner in iterative solvers.

In this talk, we consider three different FEM-BEM coupling techniques, namely the Bielak-MacCamy coupling, Costabel's symmetric coupling, and the Johnson-Nédélec coupling for transmission problems posed on unbounded domains where one equation is given on some bounded domain, and the other linear equation is posed on the complement of the bounded domain.

We observe that the inverses of stiffness matrices corresponding to the lowest order FEM-BEM discretizations can be approximated by the data-sparse format of $\mathcal{H}$-matrices. Moreover, we show that the error decays exponentially in the block rank employed [1]. We combine corresponding results for FEM-operators of [2] for elliptic problems and BEM-operators of [3] and [4] for hyper-singular and simple-layer operators, and extend the results to an abstract framework.

References:

[1] M. Faustmann, J. M. Melenk, and M. Parvizi, Caccioppoli-type estimates and $\mathcal{H}$-matrix approximations to inverses for FEM-BEM couplings, arXiv preprint arXiv:2008.11498 (2020)

[2] M. Faustmann, J. M. Melenk, and D. Praetorius, $\mathcal{H}$-matrix approximability of the inverses of FEM matrices, Numer. Math. 131 (2015), no. 4, 615–642

[3] M. Faustmann, J. M. Melenk, and D. Praetorius, Existence of $\mathcal{H}$-matrix approximants to the inverse of BEM matrices: the hyper-singular integral operator, IMA J. Numer. Anal. 37 (2017), no. 3, 1211–1244

[4] M. Faustmann, J.M. Melenk, and D. Praetorius, Existence of $\mathcal{H}$-matrix approximants to the inverses of BEM matrices: the simple-layer operator, Math. Comp. 85 (2016), no. 297, 119–152

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