Convergent finite element methods for the Ericksen model of nematic liquid crystals

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Liquid crystals (LCs) are materials which exhibit properties intermediate between isotropic liquids and crystalline solids. The Ericksen model describes nematic LCs in terms of a unit-length vector field $\boldsymbol{n}$ and a scalar function s. Equilibrium states of the LC are given by admissible pairs $(s, \boldsymbol{n})$ that minimize the energy functional $E[s, \boldsymbol{n}] = \frac{1}{2}\int_\Omega \left(\kappa|\nabla s|^2 + s^2|\boldsymbol{\nabla n}|^2\right) + \int_\Omega \psi(s)$, where $\kappa > 0$ is constant and ψ denotes the double well potential. We propose a simple but novel finite element approximation of the problem that can be implemented easily within standard finite element packages. The scheme does not employ a projection to impose the unit-length constraint on $\boldsymbol{n}$ and thus circumvents the use of weakly acute meshes, which are quite restrictive in 3D but are required by recent algorithms for convergence. We show stability and Γ-convergence properties of the new method in the presence of defects. We also discuss an effective nested gradient flow algorithm for computing minimizers that controls the violation of the unit-length constraint of $\boldsymbol{n}$. We present several simulations in 2D and 3D that document the performance of the proposed scheme and its ability to capture quite intriguing defects.

References:

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