

Multilevel adaptivity for stochastic collocation finite element methods

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This talk concerns the development of adaptive refinement algorithms for the numerical solution of partial differential equations (PDEs) with uncertain inputs. Sparse grid stochastic collocation representations of parametric uncertainty, in combination with finite element discretization in physical space, have emerged as an efficient alternative to Monte-Carlo strategies, particularly in the context of nonlinear PDE models or linear PDE problems that are nonlinear in the parameterization of the uncertainty.

We present an adaptive algorithm for computing stochastic collocation finite element approximations that employ individually tailored spatial discretizations across collocation points (we call this the multilevel stochastic collocation FEM) [1, 2]. In this algorithm, we employ hierarchical a posteriori estimates for reliable error control in computed approximations and use the associated error indicators to guide the adaptive refinement process; see [1]. The performance of the developed algorithm in numerical experiments for linear elliptic PDEs with non-affine parameterization of uncertain inputs and with parameter-dependent local spatial features will be discussed in detail. In particular, we will demonstrate the effectivity and robustness of the proposed error estimation strategy and discuss the convergence properties of the generated adaptive multilevel approximations.

References:

- [1] <https://arxiv.org/abs/2109.07320>
- [2] <https://arxiv.org/abs/2202.08902>

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